CONSUMPTION HETEROGENEITY: MICRO DRIVERS AND MACRO IMPLICATIONS

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Edmund Crawley¹  Andreas Kuchler²
Federal Reserve Board  Danmarks Nationalbank

Abstract
We document heterogeneity in the marginal propensity to consume (MPC) across household characteristics relevant to understanding heterogeneous agent models and monetary policy transmission. We find a strong negative relationship between household liquid wealth and MPC. We show that household liquid wealth predicts MPC closely for every other household characteristic we look at. We use a new empirical method that overcomes sources of bias found in the existing literature, along with administrative data from Denmark that allows us to identify heterogeneous behavior. We use our results to analyze monetary policy transmission mechanisms in both Denmark and the United States.

Keywords  Uncertainty, Consumption Dynamics, MPC
JEL codes  D12, D31, D91, E21

¹Crawley: Federal Reserve Board, Constitution Avenue & 20th Street NW, Washington, DC 20551, USA, edmund.s.crawley@frb.gov.  ²Kuchler: Danmarks Nationalbank, Langelinie Alle 47, 2100 Copenhagen Ø, Denmark, aku@nationalbanken.dk.

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1 Introduction

How does consumption behavior vary across households? Answering this question will not only lead to a better understanding of the micro drivers of household consumption-savings decisions, but can also help to quantify the macroeconomic consequences of household heterogeneity. Recent heterogeneous agent models suggest that wealth redistribution between households with high and low marginal propensity to consume (MPC) may play a dominant role in propagating macroeconomic shocks, particularly for monetary policy. Testing the microfoundations of these models empirically, and quantifying the macroeconomic importance of redistribution, often boils down to measuring how MPCs vary systematically over dimensions such as wealth and exposure to interest rate movements. However, shortcomings both in the empirical methods used to measure MPCs and in the available data have limited economists’ ability to do this reliably.

In this paper we overcome some of these shortcomings. We present a new method to measure MPCs from income and consumption panel data, building upon that of Blundell, Pistaferri, and Preston (2008) (henceforth BPP). We then apply our method to different groups of households in administrative data from Denmark, finding large differences associated with levels of household liquid wealth. We find that we can accurately predict the MPC for any group of households knowing only the group’s median level of liquid wealth. We estimate the average MPC in the economy, measured over a one-year period, to be 0.64. This estimate is in line with those obtained from natural experiments, albeit at the high end of those estimates. In contrast to our estimate, those obtained from natural experiments commonly only cover a period of three months of consumption following an income shock.

We follow BPP by imposing identifying restrictions on household income and consumption dynamics. We make improvements to the method in order to address two distinct criticisms of the BPP method. First, Crawley (2020) documents that the time aggregated nature of income and consumption data can result in biased estimates. We explicitly model the time aggregated nature of income and consumption in our data in order to avoid these biases. Second, Commault (2021) shows that when the consumption response to transitory income shocks does not follow a random walk and these income shocks have some persistence, the BPP method can again result in bias. In our model, we allow transitory income shocks to persist up to two years and allow complete freedom in how consumption responds to a transitory income shock for up to two years, after which we assume no further response. By comparison to Commault (2021), our model
allows for more freedom in the nature of transitory income shocks but imposes slightly stricter restrictions on the consumption response.

Our data consist of a panel of income and wealth for the entire Danish population. Income and wealth data are largely third-party reported to tax authorities and correspondingly accurate. Accordingly, we use the household budget constraint to back out expenditure from income and wealth which allows us to study the relation between income and spending. These Danish administrative data also include details of the interest rate sensitivity of households' financial assets and liabilities that we use to estimate the redistribution effects of monetary policy. The detail we have on households balance sheets allows us to investigate heterogeneity in consumption behavior along dimensions that have previously proved difficult to measure.

Speaking to the microfoundations of consumption behavior, we uncover a clear negative monotonic relation between MPC and liquid wealth. We show that the sign of this relationship is in line with standard buffer-stock models, although the magnitude of MPCs, especially for households with the most liquid assets, is difficult to reconcile with theory. When we include illiquid wealth, such as housing, this monotonic relationship between wealth and MPC no longer holds: Those with close to zero net worth have higher MPCs than both those with negative and positive net worth. In fact, we find liquid wealth closely predicts MPCs across all the dimensions we consider in the paper.

Broadly speaking, in our data, we see three groups with distinct MPCs and exposure to interest rate movements: the “poor hand-to-mouth,” with MPCs around 0.8, who own few assets, liquid or otherwise and are not directly exposed to interest rates; the “wealthy hand-to-mouth,” with MPCs around 0.6, who typically own houses and have mortgages and other debts whose payments rise with interest rates; and the “wealthy,” a smaller group with MPCs around 0.3, who typically own houses and also have large liquid bank balances, and whose income rises with interest rates. These groups loosely line up with those of the same name in Violante, Kaplan, and Weidner (2014), who define wealthy hand-to-mouth as households with significant illiquid assets but little or no liquid assets.

The strength of our method and data over previous studies can be seen when we quantify the size of monetary policy redistribution channels. We follow the decomposition of Auclert (2019) who reports sufficient statistics that determine the effect of monetary policy redistribution on aggregate consumption. However, being limited by the econometric methods he has at hand as well as by publicly available data sources,
he finds it challenging to get a clear picture of how MPCs vary over the dimensions he identifies as relevant, such as unhedged interest rate exposure.

We estimate that, in Denmark, a 1-percentage-point rise in the one-year real interest rate, which redistributes wealth from the wealthy hand-to-mouth debtors who pay interest to the wealthy creditors who receive it, reduces aggregate consumption growth by 29 basis points through this redistribution channel alone. We find this channel is much smaller in the U.S where, relative to Denmark, household debt is lower and the interest rate paid on mortgages adjusts less frequently.

We believe the rich detail we are able to provide on the relationships between MPC, home ownership, liquidity, and interest rate exposure could be used to discipline microfounded macroeconomic models going forward. Furthermore, a growing number of large, high-quality panel datasets on income and consumption are becoming available to economists, which increases the value of robust econometric methods that can uncover household behavior. Beyond the applications in this paper, our method has a wide variety of potential applications in the consumption, household finance, and labor literatures.

1.1 Literature Review

This paper adds to the existing literature on estimating household consumption behavior in response to income shocks. Much of the older literature focused on estimating an average MPC for the economy as a whole, while recently there has been an effort to identify heterogeneity in consumption behavior. Table I shows a selection of the population average estimates from the literature. An overview of the older literature on consumption responses to income changes is found in Jappelli and Pistaferri (2010).

The empirical methods fall into three categories.

The first category—our method falls in this category—is to impose covariance restrictions on panel data of income and consumption and use these to identify the consumption response to income shocks of differing persistence. This method has the advantage that it uses ‘typical’ income shocks for identification, as opposed to some unusual income shock such as a lottery win or a stimulus check. However, it is potentially subject to model misspecification. The most well-known paper to use this method is by Blundell, Pistaferri, and Preston (2008) (henceforth BPP), which uses imputed non-durable consumption data based on food expenditure reported in PSID data. BPP estimates a consumption elasticity (closely related to an MPC if households’ consumption level is close to their income) and find almost no consumption response to transitory
### Table I  Estimates of the Marginal Propensity to Consume from Income Shocks

<table>
<thead>
<tr>
<th>Permanent Shocks</th>
<th>Consumption Measure</th>
<th>Horizon</th>
<th>Method</th>
<th>Event/Sample</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Nondurables</td>
<td>Total PCE</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Blundell, Pistaferri, and Preston (2008)*</td>
<td>0.65</td>
<td>∼</td>
<td>3</td>
<td>Estimation Sample: 1980–92</td>
</tr>
<tr>
<td>Gelman, Gorodnichenko, Kariv, Koustas, Shapiro, Silverman, and Tadelis (2016)</td>
<td>1.0</td>
<td>∼</td>
<td>1</td>
<td>Gasoline Price Shock</td>
</tr>
</tbody>
</table>

#### Transitory Shocks

<table>
<thead>
<tr>
<th></th>
<th>Consumption Measure</th>
<th>Horizon</th>
<th>Method</th>
<th>Event/Sample</th>
</tr>
</thead>
<tbody>
<tr>
<td>Agarwal and Qian (2014)</td>
<td>0.90</td>
<td>10m</td>
<td>1</td>
<td>Singapore Growth Dividend 2011</td>
</tr>
<tr>
<td>Blundell, Pistaferri, and Preston (2008)*</td>
<td>0.05</td>
<td>∼</td>
<td>3</td>
<td>Estimation Sample: 1980–92</td>
</tr>
<tr>
<td>Browning and Collado (2001)</td>
<td>∼ 0</td>
<td></td>
<td>1</td>
<td>Spanish ECPF Data, 1985–95</td>
</tr>
<tr>
<td>Coronado, Lupton, and Sheiner (2005)</td>
<td>0.36</td>
<td>1y</td>
<td>1</td>
<td>2003 Tax Cut</td>
</tr>
<tr>
<td>Fuster, Kaplan, and Zafar (2018)</td>
<td>0.08–0.31</td>
<td>3m</td>
<td>2</td>
<td>NY Fed Survey Cons. Expectations</td>
</tr>
<tr>
<td>Hausman (2012)</td>
<td>0.6–0.75</td>
<td>1y</td>
<td>1</td>
<td>1936 Veterans’ Bonus</td>
</tr>
<tr>
<td>Hsieh (2003)*</td>
<td>∼ 0</td>
<td>0.6–0.75</td>
<td>1</td>
<td>CEX, 1980–2001</td>
</tr>
<tr>
<td>Jappelli and Pistaferri (2014)</td>
<td>0.48</td>
<td></td>
<td>2</td>
<td>Italy, 2010</td>
</tr>
<tr>
<td>Johnson, Parker, and Souleles (2009)</td>
<td>∼ 0.25</td>
<td>3m</td>
<td>1</td>
<td>2003 Child Tax Credit</td>
</tr>
<tr>
<td>Lusardi (1996)*</td>
<td>0.2–0.5</td>
<td></td>
<td>3</td>
<td>Estimation Sample: 1980–87</td>
</tr>
<tr>
<td>Parker (1999)</td>
<td>0.2</td>
<td></td>
<td>3</td>
<td>Estimation Sample: 1980–93</td>
</tr>
<tr>
<td>Parker, Souleles, Johnson, and McClelland (2013)</td>
<td>0.12–0.30</td>
<td>0.50–0.90</td>
<td>3m</td>
<td>2008 Economic Stimulus</td>
</tr>
<tr>
<td>Sahm, Shapiro, and Slemrod (2010)</td>
<td>∼ 1/3</td>
<td>1y</td>
<td>1</td>
<td>2008 Economic Stimulus</td>
</tr>
<tr>
<td>Shapiro and Slemrod (2009)</td>
<td>∼ 1/3</td>
<td>1y</td>
<td>1</td>
<td>2008 Economic Stimulus</td>
</tr>
<tr>
<td>Souleles (1999)</td>
<td>0.045–0.09</td>
<td>0.34–0.64</td>
<td>3m</td>
<td>1</td>
</tr>
<tr>
<td>Souleles (2002)</td>
<td>0.6–0.9</td>
<td>1y</td>
<td>1</td>
<td>The Reagan Tax Cuts of the Early 1980s</td>
</tr>
</tbody>
</table>

* Elasticity of consumption to income.

Methods: 1) Natural experiment 2) Survey question 3) Covariance restrictions

This table is adapted from Carroll, Slacalek, Tokuoka, and White (2017).
shocks. Both Crawley (2020) and Commault (2021) suggest misspecification problems with the methodology in BPP that bias the consumption response to transitory shocks: Crawley (2020) with respect to time aggregation and Commault (2021) with respect to income shock persistence combined with transitory consumption dynamics. In this paper, we account for both of these problems. Our methodology differs from that in Commault (2021) in that we apply a slightly stricter condition on transitory consumption dynamics and a slightly looser condition on transitory income dynamics. We compare the results from our methodology to that in Blundell, Pistaferri, and Preston (2008) and Commault (2021) in online appendices C and D respectively. We find the results using the robust estimator found in Commault (2021) are substantially in agreement with our baseline results, while those following the method from BPP look different. The BPP method, and those closely related to it, has become a standard tool in the consumption literature. See, for example, Violante, Kaplan, and Weidner (2014); Auclert (2019); and Manovskii and Hryshko (2017).

The second category of methods to estimate consumption behavior is to identify a natural experiment and measure the consumption response to it. For example, Parker, Souleles, Johnson, and McClelland (2013) use randomly assigned timing of 2008 stimulus payments and questions in the U.S. Consumer Expenditure Survey to identify a three-month aggregate marginal propensity to consume, including durables, of 0.5 to 0.9. Lewis, Melcangi, and Pilossoph (2019) use the same 2008 natural experiment and find a large amount of heterogeneity in MPC, most of it unexplained. Identification from natural experiments in this second category is less model dependent than the first category, but MPC estimates vary and there is no strong consensus. Moreover, some identification issues arise as to when exactly households learn about the payment versus when it is received, and the extent to which external validity extends from these natural experiments to the kinds of ‘typical’ transitory shocks found in heterogeneous agent models is unclear. Some of these studies find a smaller MPC for positive shocks than negative shocks—for example, Bunn, Le Roux, Reinold, and Surico (2018). In this paper we implicitly assume that the response is symmetric. In practice, our estimates represent an average of positive and negative shock reactions.

Finally, the third category involves simply asking individuals how much of a transitory income change they would consume. Jappelli and Pistaferri (2014) find an aggregate MPC of 0.48 using the Italian Survey of Household Income and Wealth and are able to identify clear differences across levels of liquid wealth. Fuster, Kaplan, and Zafar (2018) find a lower aggregate MPC in the NY Fed’s Survey of Consumer Expectations, but
they find heterogeneity by both size and sign of the shock. Of course the reliability of these studies is limited by the accuracy of households’ own response to the question.

Big data sources, such as credit and debit card transaction data and registry data of the type used in this paper, have recently allowed researchers to dig into how consumption behavior varies systematically across household characteristics. Fagereng, Holm, and Natvik (2021) finds a similar, albeit weaker, relationship between household liquid wealth and MPC using lottery winnings and Norwegian registry data on income and wealth. Credit and debit card transactions from banks or financial aggregators can be used to identify heterogeneous consumption responses at a high frequency. Baker (2018) is one of the first papers to make use of these type of data and looks at how consumption behavior varies with household debt. Other papers have focused on the relationship between liquidity and consumption smoothing: Gelman (2021) finds large differences in the impulse response to a tax rebate at a monthly frequency across household quintiles of cash-on-hand; Ganong, Jones, Noel, Greig, Farrell, and Wheat (2020) exploit a rich source of data from JPMorgan and, similar to our paper, finds differences in household liquid wealth can explain differences in consumption elasticity across a variety of other dimensions, with a focus on race. At a very high frequency, Olafsson and Pagel (2018) looks at consumption around payday in Icelandic data and finds strong responses even for households with liquid assets. Big, high frequency, spending data has also shown its value during the pandemic. Baker, Farrokhnia, Meyer, Pagel, and Yannelis (2020), Chetty, Friedman, Hendren, Stepner, and Team (2020), and Karger and Rajan (2020) all point to high MPC in the days and weeks following the first round of stimulus checks, especially for low-income or low liquid wealth households.

The method and the MPC estimates obtained in this paper can be used to discipline the growing literature on heterogeneous agent models. It is now widely recognized that the way in which macroeconomic shocks propagate can hinge not only on the average household MPC for an economy, but also on wealth transfers between households with different MPCs. In this paper, we provide new estimates for the sufficient statistics suggested in Auclert (2019) to measure the size of monetary policy transmission mechanisms. We believe these sufficient statistics to be highly informative of the mechanisms behind monetary policy, but the method calls for high quality MPC estimates across the relevant dimensions of heterogeneity. A key benefit of the Danish data is that the detailed balance sheet data gives us the ability to measure MPC heterogeneity along dimensions such as net nominal positions and unhedged interest rate exposure that have previously been difficult to measure with accuracy. Ampudia, Georgarakos, Slacalek,
Tristani, Vermeulen, and Violante (2018) also use Auclert’s framework, in this case to look at differences in monetary policy transmission between European countries, but this paper does not attempt to estimate MPCs. Patterson (2019) provides estimates for how much aggregate income fluctuations are amplified by MPC heterogeneity, using data from the PSID. Wong (2016) also takes an empirical approach to identifying how the consumption response to monetary policy shocks varies with age.

2 Empirical Strategy

We take a reduced form approach to estimate four parameters: the variance of permanent and transitory income shocks and the marginal propensity to consume out of permanent and transitory income shocks. To do so, we will make identifying restrictions on income and consumption dynamics. Specifically, we will assume that income is made up of a permanent component that moves as a random walk and a transitory component with persistence of less than two years. For consumption, we assume it responds permanently to a permanent income shock but has a short-lived response of no more than two years to a transitory income shock. Our model will be in continuous time in order to correctly account for the time aggregated nature of our data. These restrictions allow us to calculate a set of observable moments with which we can estimate the four parameters of interest using GMM.

2.1 Identifying Restrictions

Here, we describe the restrictions we impose on income and consumption dynamics that allow us to identify the variance of permanent and transitory income shocks, as well as how households respond to them.

2.1.1 Income Dynamics

Our identification of permanent and transitory income variance follows the methodology of Carroll and Samwick (1997) closely. As in their approach, we assume idiosyncratic income is composed of permanent and transitory components, where the permanent component follows a random walk and the transitory component persists for no more than two years. Our main innovation is to account for time aggregation by setting their discrete time model in continuous time and aggregating income over each year appropriately. We choose to model the level income process, rather than the log income.
process as in Carroll and Samwick (1997), because this allows more direct estimates of marginal propensities to consume. In online appendix L we get qualitatively similar results using a model of log income and expenditure.

Our model is set in continuous time where each time period represents one year. We define two independent martingale processes (possibly with jumps), \( P_t \) and \( Q_t \), where \( P_t \) will represent the flow of permanent income at time \( t \) and the change in \( Q_t \) provides the transitory impulses that generate the transitory income. We assume that for all \( s_1 > s_2 > s_3 > s_4 > 0 \):

\[
\text{Var}(P_{s_1} - P_{s_2}) = (s_1 - s_2)\sigma_P^2 \\
\text{Cov}(P_{s_1} - P_{s_2}, P_{s_3} - P_{s_4}) = 0 \\
P_s = 0 \quad \text{if } s < 0
\]

and similarly for \( Q_t \). That is, these martingales have independent increments. As a useful benchmark, two independent Brownian motions satisfy these criteria.

The natural generalization of the MA(2) transitory income process from Carroll and Samwick (1997) is to allow for a generically shaped transitory income shock that decays to zero in under two years. Figure I shows an example of such a transitory income shape \( f(t) \), but the model also allows for completely transitory shocks in which case \( f(t) \) would be a delta function with all the income from the transitory shocks arriving as a mass at the time of the shock. In this model the flow of income arriving at time \( t \) is given by the flow of permanent income and the sum of income arising from any transitory shocks to income that have occurred in the previous two years:

\[
y_t = P_t + \int_{t-2}^{t} f(t-s) dQ_s.
\]

We do not observe \( y_t \) directly but instead \( \bar{y}_T \), the time aggregated income over each one year period:

\[
\bar{y}_T = \int_{T-1}^{T} y_t dt \text{ for } T \in \{1, 2, 3...\}.
\]
Taking the $N^{th}$ difference for $N \geq 3$ we get:

$$\Delta^N \bar{y}_T = \int_{T-1}^{T} y_t dt - \int_{T-N-1}^{T-N} y_t dt$$

$$= \int_{T-1}^{T} (T - s) dP_s + (P_{T-1} - P_{T-N}) + \int_{T-N-1}^{T-N} (s - (T - 2)) dP_s$$

$$+ \left( \int_{T-1}^{T} \int_{t-2}^{t} f(t-s) dQ_t dt - \int_{T-N-1}^{T-N} \int_{t-2}^{t} f(t-s) dQ_t dt \right). \quad (2)$$

The variance of time aggregated income of an $N$ year period is therefore:\footnote{See online appendix A for full details of this derivation.}

$$\text{Var}(\Delta^N \bar{y}_T) = (N - \frac{1}{3}) \sigma_P^2 + 2\text{Var}(\tilde{y}) \text{ for } N \geq 3. \quad (3)$$

This is similar to the discrete time model in Carroll and Samwick (1997) except that the coefficient on permanent variance is $N - \frac{1}{3}$ in place of $N$. The transitory variance identified is the variance of “total” transitory income received in the year, $\bar{y}$, where this is defined as:\footnote{In the discrete time MA(2) model, $y_t = p_t + \varepsilon_t + \theta_1 \varepsilon_{t-1} + \theta_2 \varepsilon_{t-2}$, different definitions of transitory variance are used. Carroll and Samwick (1997) estimate $(1 + \theta_1^2 + 2 \theta_2^2) \sigma_\varepsilon$, while Blundell, Pistaferri, and Preston (2008) estimate $\sigma_\varepsilon$. Our definition of $\text{Var}(\tilde{y})$ is the continuous time analog of Carroll and Samwick (1997). There is no clear analog of $\sigma_\varepsilon$ in continuous time.}

$$\bar{y}_T = \int_{T-1}^{T} \int_{t-2}^{t} f(t-s) dQ_s dt. \quad (4)$$

Equation 3 shows that the variance of income growth grows linearly with the number of years of growth beyond three years. This result comes from the fact that the transitory component adds variance at the beginning and end of the growth period, but any transitory shock to income that occurs in the middle of the period does not affect income growth, as it will have decayed by the end of the measured period.

In online appendix J and K we explore alternative models of income processes, including heterogeneous income profile models and time-varying risk models.

### 2.1.2 Consumption Dynamics

Our approach will be to extend the identification of income variance by using growth over three, four and five years to also identify the covariance of income and consumption. In contrast to Blundell, Pistaferri, and Preston (2008), who assume that consumption follows a random walk, we will instead assume that the impulse response to a transitory shock follows a generic path, $g(t)$, that, like the transitory income shock, has fallen to zero two years after the news of the shock. Figure I shows possible paths for both income and consumption, along with the alternative random walk impulse response of
BPP. The best evidence for the speed at which the consumption response decays comes from Gelman (2021) and Fagereng, Holm, and Natvik (2021), both of which show that the response has entirely (or almost entirely) decayed two years after the shock. To the extent that the consumption response has not decayed after two years, our method will underestimate the size of the year-one consumption response. In online appendix I, we show that this bias is small, especially for all but the most liquid households. For permanent income shocks, we maintain the assumption from BPP that the consumption response to these shocks follows a random walk proportional to the size of the shock. Under these assumptions the instantaneous flow of consumption is given by:

\[ c_t = \phi P_s + \int_{t-2}^{t} g(t-s) dQ_s \]

and the covariance of time aggregated income and consumption growth over \( N \geq 3 \) years is given by

\[ \text{Cov}(\Delta^N \tilde{c}_T, \Delta^N \tilde{y}_T) = \phi(N - \frac{1}{3})\sigma_P^2 + 2\text{Cov}(\tilde{c}, \tilde{y}) \text{ for } N \geq 3 \] (5)

where total transitory income, \( \tilde{y} \), is given by equation 4 and total transitory consumption, \( \tilde{c} \), is defined by

\[ \tilde{c}_T = \int_{T-1}^{T} \int_{t-2}^{t} g(t-s) dQ_s dt. \] (6)
2.2 Minimum Distance Estimation

Using the equations for variance (3) and covariance (5) of observed income and consumption growth over $N$ years for at least two different values of $N$, we are able to estimate the following four unknowns in which we are interested:\footnote{We have a total of 96 moments (we have eight consecutive five-year periods, each of which has three three-year growth periods, two four-year growth periods, and one five-year growth period, $8 \times (3 + 2 + 1) = 48$. Each of these growth periods has both a variance and a covariance moment, $48 \times 2 = 96$). With only four parameters to estimate, the system is over identified. We strongly reject the null of the Sargan-Hansen J-test when run on our data, but this is not surprising given the sample size of our data.}

1. $\sigma_p^2$ Variance of permanent shocks

2. $\sigma_q^2 = \text{Var}(\tilde{y})$ Variance of transitory income received in a year

3. $\phi$ Marginal Propensity to eXpend (MPX) w.r.t. permanent income

4. $\psi = \frac{\text{Cov}(\tilde{c}, \tilde{y})}{\text{Var}(\tilde{y})}$ Regression coefficient of transitory consumption w.r.t. transitory income over a year (MPX out of transitory income).

To identify heterogeneity, we estimate these four unknowns separately for different subpopulations in our data. To the extent that there is heterogeneity within the subpopulation we estimate on, our method will provide an average transitory or permanent MPX, weighted by the variance of transitory or permanent income shocks respectively. Our panel covers 12 years of income and expenditure data and we choose to use growth over three, four and five years to balance greater identification (longer growth periods give more power) with three identification problems that grow with $N$. First, many households drop out of the sample if we demand they have reliable data for too many consecutive years. Second, if the permanent shock in fact decays slowly over time (e.g. is in fact AR(1)), the bias this introduces will be larger for large $N$. Third, the validity of running the regressions in levels (rather than logs) is reduced over large $N$ when the potential for the variance of income to change significantly from the start to the end of the sample is high. In online appendix L we test the importance of these issues.

We follow Blundell, Pistaferri, and Preston (2008) and use diagonally weighted minimum distance estimation, although our results are not significantly changed by using other popular weighting methods. In general, our results may be subject to misspecification problems, but the sample size of our data means that standard errors are small.

As the main part of our analysis will focus on the parameter $\psi$, it is worth describing exactly what this is and why we have labeled it the marginal propensity to expend out of transitory income. If we were able to exactly observe transitory income and...
consumption resulting from transitory income, then $\psi$ would be the regression coefficient of this transitory consumption on transitory income. If transitory income shocks have no persistence, this is approximately a six-month MPX (on average, the shock will happen six months into the year so that the regression will pick up the change in consumption in the following six months). If transitory income shocks have a little persistence (online appendix E shows evidence of a small amount of transitory income persistence), $\psi$ can only loosely be interpreted as the MPX to an income shock, and the reader should bear in mind that the true interpretation is, “if income is higher by one unit this year due to transitory factors, then consumption this year will be expected to be higher by $\psi$ units.”

3 Data

Our panel data on income and wealth, from which we impute expenditure, comes from Danish registry data from 2003-2015. These data have a number of advantages over survey-based measures. First, the sample contains millions of households rather than thousands. Second, households are required by law to report their data, so there is much less risk of selection bias through drop outs. Third, measurement error in income data is largely eradicated, as employees' income data is third party reported by their employer, compared to survey data where self-reported income has been shown to be particularly unreliable for irregular income.4

3.1 Income

We are interested in income and consumption decisions at the household level. We define a household as having either one or two adult members. Two adults are considered to be in the same household if they are living together and (i) are married to each other or have entered into a registered partnership, (ii) have at least one common child registered in the Civil Registration System or, (iii) are of opposite sex and have an age difference of 15 years or less, are not closely related and live in a household with no other adults. In the panel data, an individual’s household will change if he or she gets married or divorced, which leads to some selection bias given that we require households to survive for at least five years. Following the literature on consumption dynamics, our baseline results use total household labor income. In the online appendix L, we show that the results using only income from the head of household yields quantitatively similar

\[4\text{See David, Marquis, Moore, Stinson, and Welniak (1997) for a survey of income measurement error issues in survey data.}\]
results. We use after tax and transfer income, as we are interested in the consumption response to these changes in income, although the method could be used to measure the extent of consumption insurance provided by the tax and transfer system. Our data come from the administrative records from the tax authority. The tax reporting system in Denmark is highly automated and individuals bear little of the reporting burden. For employees, income is reported by their employer and is thought to be highly accurate. The underground economy in Denmark is small. We remove business owners from the sample, as their income may be less accurately reported but, more importantly, the expenditure imputation method does not work well for them (see section 3.2).

We work with the residual of income after controlling for observable characteristics of households that may affect their income and consumption. To start, we remove households in the top and bottom 1% of the income distribution. We then normalize by average household income over the observed period and regress income on dummies for age, year, highest level of education, marital status, homeowner status, and number of children along with interaction of age with education, marital status, and homeowner status. We take the change in the residuals of this regression to be the unexpected income change for a household from one year to the next.

### 3.2 Imputed Expenditure

Our expenditure data come from imputing expenditure from income and wealth. Along with other Scandinavian countries, Denmark is unusual in that tax reporting includes information about wealth along with income, a legacy from the wealth tax that was phased out between 1989 and 1997. Following the methodology from Browning and Leth-Petersen (2003) and Fagereng and Halvorsen (2017), we impute expenditure using the identity

\[
\bar{C}_t \equiv \bar{Y}_t - \bar{S}_t = \bar{Y}_t - P_t - \Delta NW_t
\]

where \( \bar{C}_t, \bar{Y}_t, \) and \( \bar{S}_t \) are the sum of expenditure, income, and savings over the year \( t \), respectively. \( P_t \) is contributions to privately administered pension schemes, for which we have accurate data due to tax deductibility, \( \Delta NW_t \) is the change in (non-pension, non-housing) net worth measured at the end of years \( t \) and \( t - 1 \). Banks and brokers are required to report the value of their clients’ accounts on December 31 each year, and the tax reporting year runs from January 1 to December 31, so the data for income and wealth reported in the tax returns match that required to use this identity to impute consumption.
Imputing expenditure in this way captures both durable and non-durable expenditure. To highlight that this measure is different to a non-durable consumption measure often used to estimate MPCs, we refer to our estimates as a Marginal Propensity to eXpend (MPX) for the rest of the paper. The inclusion of durable expenditure also has implications for the model used—in theory a permanent shock may induce a one-off splurge on durables to increase to durable stock to the desired level. In online appendix M we explore two ways to gauge how the inclusion of durables may affect our results. First, we show that including a one-off durable spending response to permanent shocks does not change our estimates, but does slightly alter their interpretation—the transitory MPX includes durable expenditure while the permanent MPX includes the permanent increase in the annual durable expenditure, but not the one-off splurge. Second, using administrative data on car ownership we estimate the size of the durable MPX component.

The imputation method works well for households with simple financial lives. However, a recognized problem with the method is its inability to handle capital gains well. The income used for expenditure imputation includes all labor income and capital income; however, it excludes capital gains. The change in the value of assets a household has from one year to the next will depend both on their savings from reported income and their capital gains and losses. We handle these capital gains and losses in a number of ways. First, we completely exclude housing wealth, treating housing as an off-balance-sheet asset for this calculation. The problem with treating housing in this way is that we must exclude households in years in which they are involved in a housing transaction. For the self-employed, it is also difficult to distinguish between expenditure and investment in their business, so we exclude all households that receive more than a trivial amount of their income from business ventures. Finally, households that hold significant equity investments are likely to see sizable capital gains and losses. We make a naive adjustment by assuming that they hold a diversified index of stocks. While this assumption will likely lead to significant measurement error for these individuals, the concern is mitigated first by the fact that stock holding is much more unusual in Denmark than in the United States. Only around 10% of households hold any stocks, and for many of those households stocks make up only a small proportion of their total wealth. Furthermore, measurement error that is uncorrelated with idiosyncratic labor income changes do not bias our results and most of the sources of capital gains and losses

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5We exclude housing wealth from net worth only for the imputation procedure, which correctly identifies expenditure for years in which there is no housing transaction—knowing house prices with precision in these years would not help impute expenditure. Our measure of net wealth where wealth itself is of interest, for example table II, includes housing.
are unrelated to a household’s labor income. Baker, Kueng, Pagel, and Meyer (2018) demonstrate this empirically in a German dataset, showing that the relation between income and imputation error is economically small.

Another concern with the imputation method is transfers of wealth—say, between family members or friends. Indeed imputed expenditure is negative for approximately 3% of households, which may explain a proportion of that. We discard both income and expenditure data for households in years in which their expenditure is negative. In online appendix L we test the robustness of our results to sample selection bias problems that these issues may give rise to.

As with income, we work with the residual of expenditure after normalizing by mean household income and controlling for the same observable features as income. We follow exactly the same steps for expenditure as those described for income in section 3.1.

Abildgren, Kuchler, Rasmussen, and Sorensen (2018) show that the mean levels of expenditure from this imputation method are close to those from the national accounts (see online appendix figure L.1). They find relatively large differences at the household level between the consumer survey and imputed expenditure, although it is not clear that this is a problem with the imputation method as opposed to the survey measure. Indeed, for car purchases, for which highly accurate register data are available, the consumer survey shows significant underreporting, consistent with Koijen, Nieuwerburgh, and Vestman (2014), who find 30% underreporting of car purchases in the Swedish consumer survey. We believe that, with the exception of transaction-level data reported by financial aggregation applications, the imputation method we use results in some of the highest quality expenditure data available to researchers for the types of questions we are addressing. Online appendix L.1 explores these measurement error issues in more detail.

3.3 Sample Selection

As our methodology requires income uncertainty to be relatively constant through the observed period and the young and old are likely to have predictable income trends unobservable to the econometrician, we limit the sample to households headed by an individual between the ages of 30 and 55 in 2008. In online appendix E we show the assumption holds for this age group. Our final sample contains 8.1 million observations from 2004 to 2015 from an age group population totaling 18.1 million. The selection criterion that reduces the sample size the most is the requirement that a household does not make a housing transaction for a period of five years. Table II shows summary
statistics for all Danish households whose head fits into this age group as a whole as well as the sample we use in the estimation. It is reassuring that both the mean and median values for after-tax income and consumption are similar in the estimation sample and the population. Our estimation sample has much lower standard deviations as a mechanical result of excluding the top and bottom 1% of the income and consumption distributions that contain extreme values. Sample selection shows up in homeownership and car ownership, as we exclude those households that buy a house at the end of a five year period but who otherwise would be counted as renters. As a result, our sample is, on average one year older than the population. Unhedged Interest Rate Exposure (URE) and Net Nominal Position (NNP) will be discussed in section 6, but again the significant differences here are due to the housing transaction criteria.

4 Income and Consumption Characteristics by Liquid Wealth

Liquidity constraints are the key microfoundation for the lack of consumption smoothing in heterogeneous agent models. In this section we look at the empirical relation between liquid wealth and the marginal propensity to expend (MPX) out of both permanent and transitory shocks to income. We find a strong monotonic negative relation. In section
5.1 we will show liquid wealth predicts MPX across all the other dimensions we look at later in this paper.

We divide our estimation sample into quintiles according to liquid wealth, where we define liquid wealth as the mean bank deposit holding over the entire sample period. The results are little changed using any other definitions of liquid wealth as long as housing and debts are excluded—see online appendix L. In online appendix E.1 we also explore how this method, which estimates an average MPX for households according to their mean liquid wealth holdings over the sample period, compares to a model in which liquid wealth holdings and MPX vary for each household from year to year.

Figure II shows the estimated income variances and MPXs for households in each quintile of liquid wealth. Looking at the left-hand variance panel first, it is noticeable that income uncertainty is highest for households in the lowest quintile of liquid wealth. This low buffer-stock, despite high income volatility, provides some evidence towards the idea that heterogeneous tastes (e.g. discount factors of risk aversion) may be more important than income risk in determining wealth held for precautionary saving. For households in the top three quintiles of liquid wealth, the similarity of their level of income risk is remarkable. Note that in contrast to standard estimates of the U.S. income process, in which transitory income variance is larger than permanent income variance, both transitory and permanent income variance in Denmark are approximately the same, likely due to the high levels of social insurance available in Denmark. The

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6For these graphs, and all similar ones in this paper, the 95% analytic confidence intervals are shown above and below each quantile estimate. Liquid wealth quintile bounds are rounded to the nearest $1,000.
variance level, at just over 0.004 for these top three quintiles, represents a standard deviation of just above 6% of permanent income per year. Note that the estimates of income variance we obtain are highly sensitive to our treatment of outliers, but in online appendix L we show our MPX estimates are not materially affected.

The right-hand panel of figure II shows our estimates for the MPX out of permanent and transitory shocks by liquid wealth quintile. The lowest wealth quintile, who hold less than $2,000 in bank deposits on average over the sample period, look somewhat like hand-to-mouth consumers. They respond almost equally to permanent and transitory shocks, spending over 80% of an income shock in the year that it arrives. However, the fact that both permanent and transitory MPXs are similar and significantly less than 1 suggests that these households may be more accurately modeled as saving in an illiquid asset such as housing or a pension following a rule of thumb (say, 20% of income) and then living hand to mouth on the remainder. As the quintile of liquid wealth increases, the MPX out of both transitory and permanent income decreases. In the top quintile, formed of households that maintained a mean bank balance above $30,000, the MPX out of permanent shocks is 0.53 and out of transitory shocks 0.32. From the point of view of theory, the responsiveness of spending out of permanent shocks in this quintile is low, while that of transitory shocks is high.

These results are broadly in line with the literature. The population mean of 0.64 for transitory MPX is a little higher than most estimates from table I, but, bearing in mind that our estimate includes durables and is best compared to a six-month to one-year MPC, it is not an outlier. The strength of the relationship between liquid wealth and MPC is similar to that found in Gelman (2021) and stronger than in Fagereng, Holm, and Natvik (2021).

### 4.1 Methodology and Result Intuition

Next we present some simple regressions of expenditure growth on income growth and compare them with what we would expect in some well understood baseline models. We have found these regressions help to build intuition on why the method, given the data, yields the results in figure II. They also help to show why the results are robust to many forms of misspecification.

We look at the estimate of $\beta^N$ in the model

$$\Delta^N c_{it} = \alpha^N + \beta^N \Delta^N y_{it} + \epsilon_{it}$$
where $N$, the number of years over which growth is measured, varies from 1 to 7. The identification of permanent and transitory MPCs in section 2.1 comes from the fact that transitory income shocks make up a relatively large proportion of the variance of income growth over a short period, while permanent income shocks dominate the variance of income growth over a long period. While the coefficients $\beta^N$ in this section do not represent MPCs, we would expect their values for large $N$ to be close to the permanent MPC, and for small $N$ to be closer to the transitory MPC.

Figure III shows what we would expect to see under three baseline models where households are subject to both transitory and permanent income shocks, as well as what we observe in the data. First, the blue horizontal line at zero shows what we would see in a complete markets model. With complete markets, all idiosyncratic shocks to income are insured against, resulting in no relation between idiosyncratic income and consumption growth. Second, the green horizontal line shows what we would see in a Solow model. Households in the Solow model do not optimize, but instead spend a constant proportion of their income each period—in this case set at 0.73—regardless of shock persistence. Third, the red, upward-sloping line shows the results for a typical buffer-stock saving model—these particular results come from the heterogeneous beta model in Carroll, Slacalek, Tokuoka, and White (2017), calibrated to match the distribution of liquid wealth in Denmark. In the buffer-stock model, the consumption response to income growth over a one-year period is small because households self-insure against the transitory shocks that dominate at this frequency. As the time period over which income growth is measured increases, the observed income growth is proportionally more permanent and self-insurance is not possible. The red line asymptotes toward 1.0 as $N$ gets large. The gray line, along with 95% confidence intervals, shows the results of these regressions using all households in the Danish sample. It is striking that the data appear to be closest to the Solow model, with only a small decrease in the regression coefficient over short periods. However, aggregating all households in this way hides a large degree of heterogeneity, particularly across households with different levels of liquid wealth.

The two black lines show the regression coefficients where the sample is restricted to households in the lowest and highest quintiles of liquid wealth (averaged over the observed period), respectively. For households in the lowest quintile there is little evidence of consumption smoothing: the consumption response to income growth over one year is both high and close to that over 7 years, strongly suggesting the MPCs for this group out of transitory and permanent shocks are similar and high. Indeed,
this is what we see in the lowest quintile of figure II—a result that is highly robust to misspecification. Households in the top quintile of liquid wealth show a clear upward slope in figure III, indicating a substantial degree of consumption smoothing. The fact that the regression coefficient for this group appears to asymptote well below 1 also suggests, in contrast to standard buffer-stock models, that the MPC out of permanent shocks for liquid households is significantly lower than 1.

5 Distribution of MPX across Net Wealth, NNP, URE, and Income

In this section we estimate the distribution of MPX along the dimensions of net wealth, Net Nominal Position (NNP), Unhedged Interest Rate Exposure (URE) and income. Having uncovered clear patterns of heterogeneity, we show that these patterns are well predicted by the liquid wealth holdings of households along these dimensions.

Figure IV shows the estimates for households grouped by quintiles of net wealth. In contrast to the monotonic relation between liquid wealth and MPX, the relation between
net wealth and MPX is humped shaped. Households in the lowest quintile of net wealth are in debt, but have a lower average MPX than households in the second quintile of net wealth, who have close to zero wealth. This may be because households with negative net wealth, such as young households with student loans, may have more access to credit and maintain a larger buffer stock of liquid assets.

Next we define two household characteristics, NNP and URE, along which MPX heterogeneity has important consequences for monetary policy:

- **Net Nominal Position (NNP)** is the net value of a household’s nominal assets and liabilities. Its relevance for analyzing the redistributive effects of monetary policy comes from the fact that an unexpected rise in the price level will decrease the real wealth of households with positive nominal assets, redistributing it to those with negative NNP (who now have less real debt). In our administrative data, we observe directly held nominal positions and their composition at the household level, including bank deposits and loans, bond holdings, and mortgages. In aggregate, the directly held NNP position of the household sector is negative, which is balanced in the national accounts by the financial sector as well as foreigners.

- **Unhedged Interest Rate Exposure (URE)** is the difference between all maturing assets (including income) and liabilities (including planned consumption). Here we define “maturing” assets and liabilities as those that are due to have their interest rates reset, even if they contractually exist for a longer period. For
example, a 30 year variable rate mortgage with annual interest rate fixation periods is “maturing” each year in this definition. Accordingly, households with a large variable rate mortgage will likely have very negative URE. For these households, the entire value of their mortgage will be adjusted to the new rate. When the interest rate rises they will see their disposable income (after mortgage payments) go down, and hence if they have a high MPX their spending will also decrease. As with NNP we find households on aggregate have a negative URE position in our data, and this is counterbalanced by the interest rate position of the financial sector. Online appendix F.1 contains relevant features of the Danish mortgage market and online appendix H shows how we calculate NNP and URE positions.

Figure V shows how the transitory MPX varies across households, according to their URE, NNP, and income. In each case, the value on the x-axis has been divided by the mean level of expenditure in the sample. The top panel shows the estimated MPX for each decile of unhedged interest rate exposure. The deciles on the left contain households most negatively exposed to a rise in interest rates, those in the middle deciles have little exposure, while the two top deciles on the right have the most to gain from an interest rate rise. We have included in this figure data on both rates of homeownership and median liquid assets for each decile. A pattern emerges in which we can roughly categorize the deciles into three groups following Violante, Kaplan, and Weidner (2014):

- **Wealthy Hand-to-Mouth**: The first five deciles contain households with high homeownership rates but few liquid assets. These households have relatively high MPXs, and it is likely that their wealth is locked up in illiquid assets (mostly housing) and that they have large mortgages.

- **Poor Hand-to-Mouth**: The next three deciles tend to be renters with little in the way of liquid assets either. These households have high MPXs and are close to being truly hand-to-mouth. As they have few assets, they have little exposure to interest rates and cannot easily substitute consumption between periods; therefore, their consumption behavior is likely not affected by changes in interest rates directly despite being highly responsive to income changes.

- **Wealthy**: The top two deciles contain households that are both likely to be homeowners and hold large liquid asset balances. These are likely to be households that own their house outright without a mortgage and have been able to build up a large stock of liquid assets. Relative to the other deciles, they have low MPX and are likely able to use their assets to effectively smooth consumption.
Figure V  MPX Distribution by URE, NNP, and Income
The distribution of MPX with NNP follows a similar pattern. This pattern has not been evident in previous attempts to measure the distribution of MPX across these dimensions.

The final chart in figure V shows the distribution of transitory MPX with total household income. There is a clear downward trend. If the income of lower-income households decreases more than that of high-income households during a monetary policy contraction, then expenditure will go down by more than the mean income-weighted MPX that would be the result of a representative agent model. For comparison, the distribution of MPX out of permanent income shocks across these three dimensions can be found in online appendix P.

5.1 Liquid Wealth Predicts MPX

In this section we show that the estimate of a group’s MPX obtained only from the median level of liquid wealth in the group is, for all practical purposes, the same as that obtained from running our full estimation procedure on that group. This allows us to plausibly extend our analysis beyond Denmark to measure the extent of MPX heterogeneity in countries for which data on liquid wealth is available, such as the United States.

We proceed by first estimating the MPX for Danish households for each decile of liquid wealth. We then take the median liquid wealth for the group of households we are interested in, and linearly interpolate an MPX based on the position of this median liquid wealth for the group within the deciles of liquid wealth whose MPX are known. For comparison, we repeat the procedure interpolating based on median income in place of liquid wealth.

Figure VI shows how estimates obtained in this way, using only the median liquid wealth (or income) for each decile, compares with the full estimation of transitory MPX for each decile of net wealth and URE. Median liquid wealth approximates the full estimation level and shape closely, while using income results in significant error. Table III shows, for each of the dimensions we have previously looked at in this paper, that median liquid wealth predicts the transitory MPX for each decile to around three percentage points, and the permanent MPX for each decile to around four percentage points. Surprisingly, the absolute level of liquid wealth is better at predicting MPX than the ratio of liquid wealth to permanent income, shown in columns 2 and 4 of table III. In contrast, using median income to predict MPX across deciles of other dimensions results in large errors.
Figure VI  Transitory MPX Interpolated From Liquid Wealth

Table III  Mean Prediction Errors Using Interpolated MPX’s

<table>
<thead>
<tr>
<th>Interpolate using:</th>
<th>Transitory MPX</th>
<th>Permanent MPX</th>
</tr>
</thead>
<tbody>
<tr>
<td>URE Deciles</td>
<td>0.03</td>
<td>0.04</td>
</tr>
<tr>
<td>NNP Deciles</td>
<td>0.03</td>
<td>0.04</td>
</tr>
<tr>
<td>Income Deciles</td>
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</tr>
<tr>
<td>Net Wealth Deciles</td>
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</tr>
</tbody>
</table>

Notes: Mean absolute errors are for the interpolated values relative to the MPX estimated using the full estimation procedure. Interpolation uses either the absolute value of liquid wealth, the ratio of liquid wealth to mean income over the sample period, or income.

Indeed, conditional on liquid wealth quintile, household income quintile has little, if any, further information on a household’s MPX. We explore this further in online appendix O.

5.2 External Validity

While there are many advantages to using data from Denmark, it is unclear to what extent the results will hold in other settings. For our analysis, the most important difference between Denmark and most other countries outside Northern Europe is that the degree of social insurance is higher in Denmark. For example, the initial unemployment benefit replacement ratios for lower income households is relatively high. This in itself reduces the need for basic consumption insurance. At the same time, labor market
Figure VII  Comparing private and non-private sector workers

regulations are more flexible than in many other countries, implying a large number of job transitions. This contributes to higher income volatility.

Though it does not allow us to abstract completely from the context, we can use the large dataset to focus on specific subgroups of the population that are arguably more comparable to households in other countries. Figure VII presents estimates of variance and MPX for households having worked at least three years in the private sector during the sample period compared to our baseline estimates. Private sector workers face larger income risk than other groups of households, and their unemployment insurance replacement ratios are lower since they have higher income than other groups. This is shown clearly in the left hand panel, in which the estimated transitory income volatility for these workers is up to two times as large as those who have not worked three years in the private sector. Similar results, not shown here, are seen for permanent income volatility. Households with private sector workers are therefore more comparable to households in other countries. In terms of external validity it is reassuring that the estimated MPX’s for private sector workers are similar to those in our baseline results.

In the following section we will use our results to assess monetary transmission in Denmark and the United States. Since our results indicate that liquid wealth is the most important dimension for the MPX we will use liquid wealth to interpolate MPX for U.S. households. The distribution of liquid wealth in Denmark and the United States is therefore important. U.S. households in the lower deciles of liquid wealth hold substantially less liquid wealth than households in the corresponding deciles in Denmark,
both in absolute terms and relative to income (see online appendix table F.1). To some extent this can be explained by the fact that credit cards are much more widely used in the United States than in Denmark. In addition, liquid wealth is measured end-of-year in Denmark (i.e. in most cases right after the monthly wage or transfer payments) whereas in the United States it is measured as an average amount over the month before the survey date. Still, it is clear that there is substantial differences between U.S. and Danish households. Therefore in online appendix N we assess the sensitivity of our results for the United States to different methods of interpolating MPX’s from the Danish data. More generally, as a growing number of panel datasets on income and consumption become available in different countries, a comparison of estimates from different settings comprises a promising topic for future research.

6 Monetary Policy and the Redistribution Channel

Auclert (2019) lays out a clear and intuitive theory as to how heterogeneity in the MPC out of transitory shocks affects the transmission mechanism of monetary policy. He identifies channels through which monetary policy can act, including those involving redistribution. He then uses this theory to identify a small set of sufficient statistics that help distinguish which of these channels are of quantitative importance.

While these statistics in theory are highly informative about the transmission mechanism of monetary policy, good data and MPC estimation methods are required to estimate them convincingly. Auclert states, “As administrative quality household surveys become available and more sophisticated identification methods for MPCs arise, a priority for future work is to refine the estimates I provide here.” We are able to bring our new MPC estimation method, along with administrative data from Denmark, in order to estimate Auclert’s sufficient statistics.

Our data have two significant advantages over previous efforts. First, our sample is large, containing most households in Denmark. Second, we have detailed balance sheet information on household NNP and URE that has been difficult to ascertain using data from previous studies.

Using the fact, established in section 5.1, that liquid wealth predicts MPX, we show how to extend the analysis to other countries. While there is no equivalent data source in the United States, and in particular expenditure data is hard to come by, we use the fact that liquid wealth is highly predictive of MPX to estimate the size of the monetary policy channels in the United States. Specifically, we use the Survey of Consumer Finances
(SCF)—which has detailed balance sheet information—to calculate households’ URE and NNP positions, along with liquid wealth. We divide the U.S. population into deciles along each dimension, and estimate each deciles’ MPX by interpolating the percentile of liquid wealth of the median household in each decile on the Danish MPX estimates by liquid wealth decile. In online appendix N we present broadly similar results based on alternative interpolation procedures.

Auclert’s method is to consider individual households’ consumption response to a monetary policy shock in which (i) the real rate of interest changes for one period by \( dR \), (ii) the price level makes a one-time change of \( dP \) and then remains at the new level, and (iii) aggregate income makes a transitory change of \( dY \). While the dynamics here are clearly stylized, and in particular lack any lag in the economy’s response, we believe such a simple experiment to be highly informative as to the relative sizes of each transmission channel.

Auclert (2019) divides the effect of monetary policy on aggregate consumption into five distinct channels:

\[
\frac{dC}{C} = \underbrace{\mathcal{M} \frac{dY}{Y}}_{\text{Aggregate Income Channel}} + \underbrace{\gamma \frac{dY}{Y}}_{\text{Earnings Heterogeneity Channel}} + \underbrace{\mathcal{E}_R \frac{dR}{R}}_{\text{Interest Rate Exposure Channel}} - \underbrace{\sigma \frac{dR}{R}}_{\text{Intertemporal Substitution Channel}}
\]

where \( \sigma \) is the elasticity of intertemporal substitution, \( \gamma \) is the elasticity of relative income to aggregate income, and the five sufficient statistics, \( \mathcal{M} \), \( \mathcal{E}_Y \), \( \mathcal{E}_P \), \( \mathcal{E}_R \), and \( S \), are measurable in the data. Further details on these statistics can be found in online appendix G. Here we are making the simplifying assumptions that \( \sigma \) and \( \gamma \) are common for all households—see Auclert (2019) for a discussion.

6.1 Results

Our estimates of the sufficient statistics for Denmark and the United States are shown in table IV. The aggregate income channel is summarized by \( \mathcal{M} \) that we estimate to be 0.55 in Denmark and 0.44 in the United States. This means that if income for all households in the economy increased by 1%, aggregate consumption growth would increase by 55 basis points (44 in the United States), broadly in line with calibrations of saver-spender models designed to fit evidence from Campbell and Mankiw (1989). The lower number

\footnote{The SCF does not contain saving data, which in the Danish data is a small part of URE, and we ignore this in our calculation of URE in the United States}
in the United States results from greater income inequality than in Denmark, and the fact that high income households (who receive most of a proportional income increase) have relatively lower MPXs.

Income inequality in the United States also results in a much larger role for the redistribution effect of income, $\varepsilon_Y$, coming from the clear negative correlation between income and MPX seen in figure V. Patterson (2019) considers this channel in the United States by dividing households into groups that have differing income sensitivity to aggregate income in PSID data. $\mathcal{S}$, the Hicksian scaling factor, is 0.47 in Denmark, which reduces the size of the intertemporal substitution channel by close to half.\(^8\)

We also see differences between Denmark and the United States in the Fisher and interest rate exposure channel, both of which are large for Denmark but less important for the United States. $\varepsilon_P$ is estimated to be negative 0.80 in Denmark, suggesting that a one-time increase in the price level of 1% increases aggregate consumption growth by 80 basis points due to redistribution from those with large nominal assets to those with large nominal debts. This Fisher channel of monetary policy is emphasized in Doepke and Schneider (2006). The fact that household debt in the United States is less than half that of Denmark contributes to the far smaller estimate of negative 0.24 for the United States. The interest rate exposure channel is also large in Denmark, while almost non-existent in the United States. We estimate $\varepsilon_R$ to be negative 0.29 in Denmark, suggesting that a 1% increase in the interest rate decreases aggregate household expenditure growth by 29 basis points. The lower prevalence of variable rate mortgages in the United States compared to Denmark contributes to a small interest rate exposure channel there. However, this does not exclude the possibility that such a channel could operate through refinancing and long-term interest rates, as highlighted in Eichenbaum, Rebelo, and Wong (2018).

7 Conclusion

In this paper we have presented a new method to measure the sensitivity of consumption to permanent and transitory income shocks for different groups of households. With administrative data from Denmark, we have been able to dig into the distribution of MPC across wealth, URE and NNP in far more detail than has previously been possible.

\(^8\)The lack of household consumption data in the SCF prevents us from estimating an equivalent statistic for the United States.
<table>
<thead>
<tr>
<th></th>
<th>$M$</th>
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<td>(0.01)</td>
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<td>(0.01)</td>
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</tbody>
</table>

**Notes:** Standard errors in parentheses. The standard errors for the Danish statistics are analytic due to the large sample size available. To calculate the standard errors for the U.S. statistics, we bootstrap using 1000 draws from the SCF data, combined with draws of the MPX for the Danish data from a normal distribution, with mean and standard deviation from the relevant MPX estimate and standard error.

We find that MPCs vary systematically with household liquid wealth, and that liquid wealth accurately predicts MPC along every other dimension we consider.

Our hope is that the method we present in this paper, or variants of it, can also be of use to economists in a variety of fields. More and more high-quality microdata on consumption are becoming available, such as the administrative data used here, or the even more detailed transaction-level data available from financial aggregators. If this trend continues, as we hope it will, methods such as ours will become even more valuable in bridging the gap between models and data.
References


Online Appendix to “Consumption Heterogeneity: Micro drivers and macro implications”, by Edmund Crawley and Andreas Kuchler

A Identification with Time Aggregation

In this section we show how to derive equations 3 and 5 for the variance of income growth and covariance of income and consumption growth.

From equation 2 we have

\[ \Delta^N \bar{y}_T = \int_{T-1}^{T} (T-s) dP_s + (P_{T-1} - P_{T-N}) + \int_{T-N}^{T-N-1} (s - (T-2)) dP_s 
+ \left( \int_{T-1}^{T} \int_{t}^{T} f(t-s) dt \right) dP_s 
+ \phi \left( \int_{T-N}^{T-N-1} \int_{t-2}^{T} (t-s) dQ_t dt \right) \]

(8)

Making use of the independent increment property of \( P_t \) and \( Q_t \), we get

\[ \text{Var}(\Delta^N \bar{y}_T) = \int_{T-1}^{T} (T-s)^2 \sigma_P^2 ds + (N-1) \sigma_P^2 \]

\[ + \phi \left( \int_{T-N}^{T-N-1} \int_{t-2}^{T} (t-s) dQ_t dt \right) \]

\[ = (N - \frac{1}{3}) \sigma_P^2 + 2 \text{Var}(\bar{y}) \text{ for } n \geq 3 \] 

(9)

The equivalent of equation 2 for consumption is

\[ \Delta^N \bar{c}_T = \int_{T-1}^{T} (T-s) \phi dP_s + \phi (P_{T-1} - P_{T-N}) + \int_{T-N}^{T-N-1} (s - (T-2)) \phi dP_s 
+ \left( \int_{T-1}^{T} \int_{t-2}^{T} f(t-s) dt \right) \phi dP_s \]

(10)

Again making use of the independent increment property, we can calculate the covariance of income and consumption growth:

\[ \text{Cov}(\Delta^N \bar{c}_T, \Delta^N \bar{y}_T) = \int_{T-1}^{T} (T-s)^2 \phi \sigma_P^2 ds + \phi (N-1) \sigma_P^2 + \int_{T-N}^{T-N-1} (s - (T-2)) \phi \sigma_P^2 ds 
+ \text{Cov} \left( \int_{T-1}^{T} \int_{t-2}^{T} f(t-s) dt, \int_{T-1}^{T} \int_{t-2}^{T} g(t-s) dQ_t dt \right) \]
\[ + \text{Cov} \left( \int_{T-N-1}^{T-N} \int_{t-2}^{t} f(t-s) dQ_s dt, \int_{T-N-1}^{T-N} \int_{t-2}^{t} g(t-s) dQ_s dt \right) \]
\[ = \phi (N - \frac{1}{3}) \sigma_p^2 + 2\text{Cov}(\tilde{c}, \tilde{y}) \text{ for } N \geq 3 \] (11)

B A Brief Introduction to the Time Aggregation Problem

An obvious question is why we have chosen not to use the well-known methodology of Blundell, Pistaferri, and Preston (2008), who achieve identification of transitory shocks from the facts that: (i) transitory income shocks in period \( t \) will mean-revert in period \( t + 1 \); and (ii) permanent shocks in period \( t \) are uncorrelated with income changes in period \( t + 1 \).\(^9\) Unfortunately, as noted in Working (1960), in time aggregated data (ii) does not hold—a time aggregated random walk is positively autocorrelated. While macroeconomists have long been aware of the importance of time aggregation in time series regressions (see Campbell and Mankiw (1989) for a well-known example), the problem has been overlooked by the household finance and labor economics literature.\(^10\) We will therefore briefly describe the problem here. For a more detailed account with particular attention to BPP, see Crawley (2020).

Time aggregation transforms an underlying, high-frequency time series, into a lower frequency time series. For example, we observe Danish tax returns at an annual frequency, while income in fact consists of paychecks arriving at a monthly, biweekly or irregular frequency. The observable income is the sum of all the income that was received by a household during the year. The key insight of Working (1960) is that even if there is no correlation between changes in income at the underlying frequency (it is a random walk), changes in the resulting time aggregated series will show positive autocorrelation. Figure B.1 shows how this autocorrelation is generated. The solid line shows the flow of income for a household that receives a permanent pay rise from $50,000 to $100,000 midway through the second year. The crosses show the income we would observe in annual tax data. During the second year, the household receives an annual $50,000 salary for six months, followed by $100,000 in the second six months, resulting in a reported income of

\(^9\)Kaplan and Violante (2010) show in discrete time simulations that the methodology works reasonably well for standard calibrations of buffer-stock models and end up concluding, “The BPP insurance coefficients should become central in quantitative macroeconomics.” However, some recent papers such as Commault (2021) and Hryshko and Manovskii (2018) have pointed to other potential problems of the methodology.

\(^10\)For examples, see Moffitt and Gottschalk (2012); Meghir and Pistaferri (2004); Nielsen and Vissing-Jorgensen (2004); Heathcote, Perri, and Violante (2010); and more recent quantile regression approaches such as Arellano, Blundell, and Bonhomme (2017).
$75,000 for the entire year. The single shock to income therefore appears in the observed data as two increases. In this way, if the underlying income process follows a random walk, shocks in one year result in observed income changes in that and the following year. The resulting autocorrelation—if the underlying process is in continuous time—is 0.25. Continuous time is a good approximation for quarterly or monthly underlying processes. The autocorrelation of an annual-time-aggregated quarterly random walk is 0.23. At a monthly or higher frequency the autocorrelation is almost indistinguishable from 0.25.\footnote{If all permanent shocks to income occurred on January 1 each year, then this would not hold. Low, Meghir, and Pistaferri (2010) show that a significant portion of permanent income variance is explained by job mobility, which can occur at any point in the year.}

While it would be possible to stick closely to the original BPP model and adjust the covariance restrictions to take account of the time aggregation problem,\footnote{Crawley (2020) takes this more straightforward approach using the same PSID data as used in BPP.} we have found that in practice the underlying assumptions made by BPP (in particular that consumption follows a random walk) do not fit with the data. The random walk assumption was previously thought to be benign. Not only were the estimates of the consumption response to transitory shocks in BPP small and consistent with such an assumption, Kaplan and Violante (2010) show that without time aggregation, the BPP method correctly identifies the transitory consumption response in the period of
the income shock regardless of the consumption dynamics going forward. This fact is again not robust to the time aggregation problem. With time aggregation taken into account, the estimates are highly sensitive to assumptions about short-term consumption dynamics—online appendix C shows how the random walk assumption affects MPX estimates. Therefore we have chosen to attain identification in a manner similar to Carroll and Samwick (1997), which allows us to be agnostic about the exact short-term dynamics of income and consumption.

C Comparison to Blundell, Pistaferri, and Preston (2008)

In this section we provide MPX estimates using the Danish data derived from the original estimation method from BPP. The estimates are shown in figure C.1. These results look different to the baseline results of this paper: the BPP transitory estimates do not vary over liquid wealth quintile and the BPP permanent estimates are high, well above one in the lowest liquid wealth quintile.

Here we will show that different models of income and consumption dynamics, all plausible, can give rise to different MPX estimates under the BPP methodology. Note

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13As in our baseline, we perform the estimation in levels, not logs, so that we can interpret our results as MPX provided the variance of permanent income change over the sample period is not large.
that the consumption data in the Danish administrative data is, like the income data, time-aggregated over the year. This contrasts with the consumption data in the PSID which provides a snapshot of consumption in the weeks before the survey is taken. The calculations we present below are therefore different to those found in Crawley (2020). These differences again serve to highlight the sensitivity of the BPP method to differences in the exact timing of income and consumption dynamics.

We will present three models of income and consumption dynamics: one in which consumption follows a random walk, one in which the consumption response to transitory shocks is transitory, and one in which there is a durables response to permanent shocks.

1) Random walk consumption response. Here we show the estimates that the BPP methodology will produce, assuming the underlying model is exactly the same as in BPP, except for the fact that shocks are distributed uniformly through the calendar year. This is the same exercise carried out in Crawley (2020), but the formulae differ due to the timing of consumption in the Danish data. The true parameters are denoted \( \sigma_P^2, \sigma_Q^2, \phi, \psi \), while the BPP estimates are denoted \( \hat{\sigma}_P^{2,BPP}, \hat{\sigma}_Q^{2,BPP}, \hat{\phi}_{BPP}, \hat{\psi}_{BPP} \).

The observed, time-aggregated, moments are:

\[
\text{Cov}(\Delta \bar{y}_T, \Delta \bar{y}_{T+1}) = \frac{1}{6} \sigma_P^2 - \sigma_Q^2 = -\hat{\sigma}_Q^{2,BPP}
\]

\[
\text{Var}(\Delta \bar{y}_T) = \frac{2}{3} \sigma_P^2 + 2 \sigma_Q^2 = \hat{\sigma}_P^{2,BPP} + 2 \hat{\sigma}_Q^{2,BPP}
\]

\[
\text{Cov}(\Delta \bar{c}_T, \Delta \bar{y}_{T+1}) = \frac{1}{6} \phi \sigma_P^2 - \frac{1}{2} \psi \sigma_Q^2 = -\hat{\psi}_{BPP} \hat{\sigma}_Q^{2,BPP}
\]

\[
\text{Cov}(\Delta \bar{c}_T, \Delta \bar{y}_T) = \frac{2}{3} \phi \sigma_P^2 = \hat{\phi}_{BPP} \hat{\sigma}_P^{2,BPP} + \hat{\psi}_{BPP} \hat{\sigma}_Q^{2,BPP}
\]

These equations can be used to derive the BPP estimate in terms of the underlying parameters:

\[
\hat{\sigma}_P^{2,BPP} = \sigma_P^2
\]

\[
\hat{\sigma}_P^{2,BPP} = \sigma_Q^2 - \frac{1}{6} \sigma_P^2
\]

\[
\hat{\phi}_{BPP} = \frac{5}{6} \phi - \frac{1}{2} \psi \frac{\sigma_Q^2}{\sigma_P^2}
\]

\[
\hat{\psi}_{BPP} = \frac{1}{\frac{1}{2} \psi \sigma_Q^2 - \frac{1}{6} \phi \sigma_P^2}{\sigma_Q^2 - \frac{1}{6} \sigma_P^2}
\]

At \( \sigma_P^2 = 1, \sigma_Q^2 = 1, \phi = 0.8, \psi = 0.8 \), this recovers \( \hat{\phi}_{BPP} = 0.27 \) and \( \hat{\psi}_{BPP} = 0.32 \). That is, if the true model is a random walk the BPP methodology underestimates both the permanent and transitory response coefficients.
2) Transitory consumption response to transitory income shocks. In this model, we assume the response to transitory income shocks is itself transitory and does not have any persistence. This close to what we find in our baseline estimates for the lowest quintile of liquid wealth. Under this model the moments are:

\[
\text{Cov}(\Delta \bar{y}_T, \Delta \bar{y}_{T+1}) = \frac{1}{6} \sigma_P^2 - \sigma_Q^2 = -\hat{\sigma}_{Q,BPP}^2 \\
\text{Var}(\Delta \bar{y}_T) = \frac{2}{3} \sigma_P^2 + 2\sigma_Q^2 = \hat{\sigma}_{P,BPP}^2 + 2\hat{\sigma}_{Q,BPP}^2 \\
\text{Cov}(\Delta \bar{c}_T, \Delta \bar{y}_{T+1}) = \frac{1}{6} \phi \sigma_P^2 - \psi \sigma_Q^2 = -\hat{\psi}_{BPP} \hat{\sigma}_{Q,BPP}^2 \\
\text{Cov}(\Delta \bar{c}_T, \Delta \bar{y}_T) = \frac{2}{3} \phi \sigma_P^2 + 2\psi \sigma_Q^2 = \hat{\phi}_{BPP} \hat{\sigma}_{P,BPP}^2 + \hat{\psi}_{BPP} \hat{\sigma}_{Q,BPP}^2
\]

Leading to BPP estimates:

\[
\hat{\sigma}_{P,BPP}^2 = \sigma_P^2 \\
\hat{\sigma}_{P,BPP}^2 = \sigma_Q^2 - \frac{1}{6} \sigma_P^2 \\
\hat{\phi}_{BPP} = \frac{5}{6} \phi + \psi \frac{\sigma_Q}{\sigma_P} \\
\hat{\psi}_{BPP} = \frac{\psi \sigma_Q^2 - \frac{1}{6} \phi \sigma_P^2}{\sigma_Q^2 - \frac{1}{6} \sigma_P^2}
\]

At \(\sigma_P^2 = 1\), \(\sigma_Q^2 = 1\), \(\phi = 0.8\), \(\psi = 0.8\), this recovers \(\hat{\phi}_{BPP} = 1.47\) and \(\hat{\psi}_{BPP} = 0.8\). That is, under this model with parameters similar to those found for the least liquid group, the BPP method significantly overestimates the consumption response to permanent shocks, while correctly estimating the consumption response to transitory shocks.

3) Durable splurge in response to permanent income shocks. In this final model, we make the same assumptions about the transitory consumption response as in (2), but further assume that households make a one-off splurge on durable goods when their permanent income rises. This is what a simple theory would tell us households should do—they need to immediately raise their stock of durables to the new higher level—and our baseline model is robust to this type of behavior (see online appendix M.1). We denote this splurge as \(\phi_d\). The observed moments are now:

\[
\text{Cov}(\Delta \bar{y}_T, \Delta \bar{y}_{T+1}) = \frac{1}{6} \sigma_P^2 - \sigma_Q^2 = -\hat{\sigma}_{Q,BPP}^2 \\
\text{Var}(\Delta \bar{y}_T) = \frac{2}{3} \sigma_P^2 + 2\sigma_Q^2 = \hat{\sigma}_{P,BPP}^2 + 2\hat{\sigma}_{Q,BPP}^2 \\
\text{Cov}(\Delta \bar{c}_T, \Delta \bar{y}_{T+1}) = \frac{1}{6} \phi \sigma_P^2 + \frac{1}{2} \phi_d \sigma_P^2 - \psi \sigma_Q^2 = -\hat{\psi}_{BPP} \hat{\sigma}_{Q,BPP}^2
\]

41
\[
\text{Cov}(\Delta \bar{v}_T, \Delta \bar{y}_T) = \frac{2}{3} \phi \sigma_p^2 + 2 \psi \sigma_Q^2 = \hat{\phi}_{\text{BPP}} \sigma_{P,\text{BPP}}^2 + \hat{\psi}_{\text{BPP}} \sigma_{Q,\text{BPP}}^2
\]

Leading to BPP estimates:
\[
\begin{align*}
\sigma_{P,\text{BPP}}^2 &= \sigma_P^2 \\
\sigma_{Q,\text{BPP}}^2 &= \sigma_Q^2 - \frac{1}{6} \sigma_P^2 \\
\hat{\phi}_{\text{BPP}} &= \frac{5}{6} \phi + \frac{1}{2} \phi_d + \psi \frac{\sigma_Q^2}{\sigma_P^2} \\
\hat{\psi}_{\text{BPP}} &= \frac{\psi \sigma_Q^2 - \frac{1}{6} \phi \sigma_P^2 - \frac{1}{2} \phi_d \sigma_P^2}{\sigma_Q^2 - \frac{1}{6} \sigma_P^2}
\end{align*}
\]

At \(\sigma_P^2 = 1, \sigma_Q^2 = 1, \phi = 0.8, \psi = 0.8, \) and \(\phi_d = 0.5\), this recovers \(\hat{\phi}_{\text{BPP}} = 1.72\) and \(\hat{\psi}_{\text{BPP}} = 0.5\), close to that obtained empirically for the households in the lowest quintile of liquid wealth.

The three models above show that the BPP model can recover estimates far removed from the underlying parameters of the model following plausible changes to the model. The third model is one way to rationalize the high permanent and low transitory MPX BPP estimate we see for households in the lowest quintile of liquid wealth. If households in higher quintiles of liquid wealth purchase relatively fewer durable goods following a permanent income shock, this may explain the low correlation observed in the BPP estimates between liquid wealth and MPX.

A further set of questions arises as to how different the Danish data is the the PSID data. Unfortunately, our method is not directly applicable to the PSID data as we assume both time-aggregated income and consumption. To apply a similar method to the PSID data would require us to make far stricter assumptions about the path of consumption following a transitory shock. Better understanding the persistence of transitory income and consumption dynamics is ongoing work by the authors.

D Comparison to Commault (2021)

Commault (2021) suggests a robust estimator for the consumption response to transitory income shocks, using the change in income from \(T + t\) to \(T + t + 1\) as an instrument for transitory shocks in period \(T\). She shows that when transitory income shocks have no persistence then change with \(t = 0\) provides an unbiased estimate even when consumption does not follow a random walk. This was also shown in Kaplan and Violante (2010). However, when transitory income shocks have persistence this is no longer true.
For persistence that lasts no more than one year, \( t = 1 \) provides an unbiased estimate, and for persistence that lasts between one and two years, \( t = 2 \) provides an unbiased estimate. These estimates with \( t \geq 1 \) are less subject to the time aggregation problem as the correlation in time-aggregated permanent shocks only persists for one year after the shock. See below for a brief discussion of time-aggregation in the context of Commault (2021).

In the PSID data, Commault (2021) finds \( \text{Cov}(\Delta \bar{y}_T, \Delta \bar{y}_{T+3}) \) to be statistically insignificant and therefore suggests \( \frac{\text{Cov}(\Delta \bar{y}_T, \Delta \bar{y}_{T+2})}{\text{Cov}(\Delta \bar{y}_T, \Delta \bar{y}_{T+2})} \) as a robust estimator for the transitory consumption response (\( t = 1 \)). In the Danish data, we find \( \text{Cov}(\Delta \bar{y}_T, \Delta \bar{y}_{T+3}) \) is statistically significant, and therefore we show results for both \( t = 1 \) and \( t = 2 \). The results are shown in figure D.1. In contrast the the BPP estimates, the results using the method from Commault (2021) are roughly in line with the baseline results from our paper. Furthermore, the estimates when \( t = 1 \) are somewhat lower than those in the baseline, which is what you would expect if there was some persistence in the transitory income shock beyond one year, which is implied by the statistical significance of \( \text{Cov}(\Delta \bar{y}_T, \Delta \bar{y}_{T+3}) \).
D.1 Time Aggregation in Commault (2021)

The Commault robust estimator is relatively robust to time-aggregation bias in data where both income and consumption is time-aggregated, such as the Danish administrative data used in this paper. For data, such as the PSID, in which income is time-aggregated but consumption is a snapshot around the time of the survey, changes to the transitory income dynamics can induce a large bias. In Commault (2021), it is assumed that transitory income arrives in two discrete lump sums, exactly one year apart. This assumption results in no time aggregation bias. Under an alternative assumption, that a transitory income shock increases the stream of income arriving for a period of one year, the response to transitory income shocks can be biased upward. For this alternative assumption, the relevant model moments are (ignoring permanent shocks and setting variance equal to one):

\[
\text{Cov}(\Delta y_{T}, \Delta y_{T+2}) = \int_{T}^{T+1} (1 - s)dQ_s \int_{T}^{T+1} -sdQ_s \\
= \int_{T}^{T+1} -s(1 - s)ds \\
= -\frac{1}{6}
\]

\[
\text{Cov}(\Delta c_{T}, \Delta y_{T+2}) = \int_{T}^{T+1} \psi dQ_s \int_{T}^{T+1} -sdQ_s \\
= \int_{T}^{T+1} -\psi ds \\
= -\frac{1}{2}\psi
\]

So the robust estimator gives \( \frac{\text{Cov}(\Delta c_{T}, \Delta y_{T+2})}{\text{Cov}(\Delta y_{T}, \Delta y_{T+2})} = 3\psi \). This upward bias may explain why the estimate in Commault (2021) is more than twice as large as that in Crawley (2020).

E Sample Selection

We choose to look at households whose head is between the ages of 30 and 55 in 2008, which is driven by the desire to remove households for which the assumption that most of the income growth is unexpected is not likely to be fulfilled. For the old and the young, individual households will likely have a lot of information about their income path that is not available to the econometrician (for example, the year in which they plan to retire, or the fact that they are on a specific career track with set expectations of promotion and
pay raises). We also want to remove households whose income volatility is increasing or decreasing sharply. Figures E.1 and E.2 show how our estimates of both income variance and MPX vary with age. The dots represent the point estimate for each age, while the lines are the centered moving averages over the five nearest age groups. The solid black line shows the total variance of income growth over one year. It should not be surprising that income growth for households with heads in their 20’s is highly volatile. This volatility plateaus around the age of 35 and stays at a constant level until retirement, at which point it temporarily grows before falling to an even lower level. We can see that while both transitory and permanent shocks to income are high early in life, permanent income shocks are particularly high while individuals find their place in the workforce. From the ages of 30 to 55, both transitory and permanent shocks are approximately the same size and remarkably stable. At retirement, shocks to permanent income rise—not surprising, as the model sees retirement itself as a shock—even as transitory income variance declines.

As the model assumes the variance to permanent and transitory shocks to be constant in the observed period, interpretation of the numbers outside of the 30 to 55 age group needs to be treated with care. However, the figure clearly shows that within this age group the assumption of constant variance appears to be a reasonable one.

The dotted black line shows the variance of $\Delta y$, assuming no persistence in the transitory component. The fact that this line is slightly above the empirical variance of $\Delta y$ is consistent with some persistence in the transitory component of income, justifying our decision to exclude growth over one and two years in our identification.

The level of both permanent and transitory shock variance for households aged 30 to 55 is approximately 0.0035, reflecting a standard deviation of 6%. Estimates using U.S. data are significantly higher, especially for the transitory shock variance (for example, Carroll and Samwick (1997) estimate 0.02 for permanent and 0.04 for transitory). This difference may be due to lower income inequality in Denmark, more progressive taxation, and more generous unemployment insurance. The lower transitory variance will also be due to significantly reduced measurement error relative to the survey-based U.S. data.

### E.1 Quantile Selection

Throughout the paper, we first select a group of households by a characteristic such as income or liquid wealth quantile, and then we estimate the parameters for that group under the assumption they are constant over time. We select quantiles according to the mean level of liquid wealth/income/etc over the sample period. However, we may be
Figure E.1  Permanent and Transitory Shock Variance by Age
Figure E.2  MPX by Age
Table E.1  Short- and Long-term Liquid Wealth Quintile Probabilities

<table>
<thead>
<tr>
<th>Short-term Quintile</th>
</tr>
</thead>
<tbody>
<tr>
<td>1   2   3   4   5</td>
</tr>
<tr>
<td>1 66.45 30.22 3.11 0.21 0.00</td>
</tr>
<tr>
<td>Long-term Quintile</td>
</tr>
<tr>
<td>2 18.15 45.18 31.21 5.10 0.35</td>
</tr>
<tr>
<td>3  6.95 17.56 42.66 29.49 3.34</td>
</tr>
<tr>
<td>4  2.93  6.25 19.13 49.18 22.51</td>
</tr>
<tr>
<td>5  0.97  1.77  4.93 18.93 73.40</td>
</tr>
</tbody>
</table>

Notes: Each row shows the percentage of time a household of that long run quintile will spend in the respective short-run quintile

Table E.2  Short- and Long-term Income Quintile Probabilities

<table>
<thead>
<tr>
<th>Short-term Quintile</th>
</tr>
</thead>
<tbody>
<tr>
<td>1   2   3   4   5</td>
</tr>
<tr>
<td>1 81.92 17.67 0.40 0.00 0.00</td>
</tr>
<tr>
<td>Long-term Quintile</td>
</tr>
<tr>
<td>2 11.12 72.16 16.32 0.39 0.02</td>
</tr>
<tr>
<td>3  0.51 10.36 66.73 20.94 1.46</td>
</tr>
<tr>
<td>4  0.01  0.48 14.59 65.79 19.13</td>
</tr>
<tr>
<td>5  0.00  0.06  1.34 17.15 81.46</td>
</tr>
</tbody>
</table>

Notes: Each row shows the percentage of time a household of that long run quintile will spend in the respective short-run quintile

interested not in the MPX for households with the lowest average level of liquid wealth over the sample period, but instead in those with the lowest level of liquid wealth in a particular year. As households may move in and out of quantiles each year, these two measures are not identical and using one as a proxy for the other may introduce bias. We label the division into quantiles according to the average over the sample period as the "long-term quantile" and the division into quantiles at any particular snapshot in time as the "short-term" quantile and show how to quantify the difference between the two.

First, we empirically measure how often transitions between the quantiles occur over the sample period. Table E.1 shows for each long-term quantile, the fraction of years each household spends in each short-term quantile, on average. For example, households
that are in the highest quintile of average liquid wealth over the sample period spend, on average, 74 percent of years in the highest quintile of liquid wealth for that year, 19 percent of years in the second highest, and just a small fraction of years in lower quintiles. Somewhat surprisingly, of all the characteristics we look at, liquid wealth is the least ‘sticky’—for other characteristics such as income (shown in table E.2) there is much less movement between quintiles. We then simulate a simple model to give an idea of how much bias is introduced by using the long-term quintiles as a proxy for the short-term quintiles. In our simple simulation, we assume that when a household is in a short-term quintile it has both the variance and the MPX for that quintile. For each long-term quintile, we then use the data from table E.1 to simulate transitions between short-term quintiles, assuming each period’s short-term quintile is chosen independently. Using this simulated data, we estimate the MPX for the long-term quintile using the method described in the paper.

Figure E.3 shows the results of this simulation. The variance of the short-term quintiles comes from the estimated values of the paper’s baseline, while the short-term MPX are chosen such that the estimated long-term MPX are close to that of the baseline. The
qualitative pattern of declining MPX with liquid wealth quintile is the same for both long- and short-term quintiles. However, the slope of the long-term quintile MPX is shallower than that of the short-term quintiles. This is a direct result of the fact that households who are in the top and bottom quintiles of long-term liquid wealth will not spend all their time in the top and bottom quintiles of short-term liquid wealth, so their MPX is shifted up and down respectively. The difference is most pronounced for the highest liquid wealth quintile.

A steeper slope would increase the size of the Auclert redistribution channels of monetary policy, so this exercise points toward the estimates we provide in the paper being a lower bound. However, such an exercise assumes a causal relationship between liquid wealth and MPX. It may be instead that the type of household that keeps a low balance of liquid wealth over a longer period is also the type of household that has a high MPX out of transitory income shocks. Furthermore, in practice it makes little difference to the results. Therefore, we have chosen not to include this analysis as part of our main results.

F Danish Household Balance Sheets

Table F.1 shows a comparison of liquid wealth and income in Denmark and the United States. Danish households on average hold more liquid wealth, both in absolute terms and as a percentage of their income.

F.1 The Danish Mortgage Market

Mortgage loans in Denmark are issued by specialized mortgage banks, which fully finance loans by issuing bonds. Interest rates are directly determined by sales prices at the bond market. That is, borrowers only pay the bond market interest rate plus a supplementary fee for the mortgage bank. Most loans are issued as 20- or 30-year loans, and households can only obtain loans from mortgage banks for up to 80% of the value at loan origination of properties used as permanent residences. The remaining (more insecure) part of the funding may be provided by commercial banks. The close link between loans and bonds, as well as fixed loan-to-value ratios, fast foreclosure procedures, full recourse, etc., mean that mortgage banks do not assume significant market risks. The status of Danish
<table>
<thead>
<tr>
<th>Liquid wealth decile</th>
<th>Liquid wealth, USD</th>
<th>Income, USD</th>
<th>Liquid wealth, % of income</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>U.S.</td>
<td>DK</td>
<td>U.S.</td>
</tr>
<tr>
<td>1</td>
<td>15</td>
<td>43</td>
<td>24,303</td>
</tr>
<tr>
<td>2</td>
<td>300</td>
<td>1,120</td>
<td>29,366</td>
</tr>
<tr>
<td>3</td>
<td>800</td>
<td>2,080</td>
<td>37,468</td>
</tr>
<tr>
<td>4</td>
<td>1,500</td>
<td>3,358</td>
<td>48,607</td>
</tr>
<tr>
<td>5</td>
<td>2,950</td>
<td>5,267</td>
<td>55,695</td>
</tr>
<tr>
<td>6</td>
<td>4,900</td>
<td>8,232</td>
<td>74,935</td>
</tr>
<tr>
<td>7</td>
<td>8,000</td>
<td>13,063</td>
<td>84,049</td>
</tr>
<tr>
<td>8</td>
<td>14,100</td>
<td>21,542</td>
<td>93,163</td>
</tr>
<tr>
<td>9</td>
<td>30,500</td>
<td>38,711</td>
<td>117,466</td>
</tr>
<tr>
<td>10</td>
<td>90,900</td>
<td>94,137</td>
<td>193,414</td>
</tr>
</tbody>
</table>

**Notes:** The table shows medians of liquid wealth (measured in USD), pre-tax annual income and liquid wealth as a percentage of pre-tax income. Data for the US is based on the Survey of Consumer Finances for 2016, and the measure of liquid wealth refers to the average balance of liquid wealth over the month before the survey date. Data for Denmark is averages of real values over 2009-2015 and refer to the balance at the end of the year. Only households aged between 30 and 55 years are included.
covered mortgage bonds as a safe asset class (AAA-rated by, e.g., S&P) implies that borrowers have access to cheap real estate funding.

The Danish mortgage system has been functioning for two centuries, but significant liberalization has taken place over the past 20 years. Variable interest loans were (re-)introduced in 1996, while interest only loans were introduced in 2003. These new loan characteristics are by now popular; see figure F.1. In contrast to the United States, where most mortgage debt is fixed rate, 40% of mortgage debt in Denmark is variable rate, with interest fixation periods mostly between six months and five years. Fixed-rate loans come with an option for early redemption, which implies that in practice, refinancing of fixed-rate mortgages often takes place, both when interest rates decrease and increase. The latter may be attractive because borrowers have the option to repay their loan by purchasing the corresponding amount of bonds. When interest rates increase, the bond value decreases, so the option to repay the loan by purchasing the corresponding amount of bonds in essence acts as an equity insurance.

Around one-fourth of the total loan balance is due to have interest rates reset over a 12-month period (see figure F.2). This figure only comprises loans that automatically will have a new interest rate and not active decisions to refinance or extract equity.

G Details on the Auclert Statistics

Table G.1 defines the five statistics calculated for Denmark in the paper.

In figure V, we estimated MPX’s for households with heads between the ages of 30 and 55, excluding the young and the old. Furthermore, some of the URE and NNP...
exposures are held indirectly on households’ balance sheets through pension funds and corporations, or by the government and foreigners, so that the URE and NNP exposure in our sample does not aggregate to zero. We allocate the aggregate URE and NNP exposure from our sample into seven bins so that the total exposure across the economy is zero. These bins include households with (i) young (<30) and (ii) old (>55) heads, and exposures held by households indirectly through (iii) pension funds, (iv) government, (v) nonfinancial corporates, (vi) financials, and (vii) exposures held by the rest of the world. Within each of these bins we assume no heterogeneity so that the MPX with respect to these exposures is constant. This assumption is conservative, and likely underestimates the size of the heterogeneous agent channels. Our assumptions on the level of these MPXs can be seen in table G.2.
<table>
<thead>
<tr>
<th>Statistic</th>
<th>Definition</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mathcal{M}$</td>
<td>$\frac{1}{C} \left[ \sum_{i \in \text{Income deciles}} MPX_i Y_i + \sum_{j \in {\text{young, old}}} MPX_j Y_j \right]$</td>
<td>Income-weighted MPX</td>
</tr>
<tr>
<td>$\mathcal{E}_Y$</td>
<td>$\mathcal{M} - \frac{\mathcal{M} \mathcal{Y}}{\mathcal{Y}}$</td>
<td>Redistribution elasticity for Y</td>
</tr>
<tr>
<td>$\mathcal{E}_P$</td>
<td>$\frac{1}{C} \left[ \sum_{i \in \text{NNP deciles}} MPX_i NNP_i + \sum_{j \in \text{bins}} MPX_j NNP_j \right]$</td>
<td>Redistribution elasticity for P</td>
</tr>
<tr>
<td>$\mathcal{E}_R$</td>
<td>$\frac{1}{C} \left[ \sum_{i \in \text{URE deciles}} MPX_i URE_i + \sum_{j \in \text{bins}} MPX_j URE_j \right]$</td>
<td>Redistribution elasticity for R</td>
</tr>
<tr>
<td>$\mathcal{S}$</td>
<td>$1 - \frac{1}{C} \left[ \sum_{i \in \text{Consumption deciles}} MPX_i C_i + \sum_{j \in {\text{young, old}}} MPX_j C_j \right]$</td>
<td>Hicksian scaling factor</td>
</tr>
</tbody>
</table>

Note: MPX is the mean MPX over all households in the economy. $Y$ and $C$ are aggregate household income and consumption respectively. Bins refer to the seven categories for which we have allocated URE and NNP exposures outside our estimation sample. $\{\text{young, old}\}$ are the two bins that contain young and old households (the other five bins are only relevant for URE and NNP exposures as $Y$ and $C$ measure household income and consumption).

We define $\mathcal{E}_R$ as

$$\mathcal{E}_R = \frac{1}{C} \left[ \sum_{i \in \text{URE deciles}} MPX_i URE_i + \sum_{j \in \text{bins}} MPX_j URE_j \right]$$  \hspace{1cm} (12)$$

where $i$ sums over the 10 deciles of URE, $j$ over the seven bins defined above, and $C$ is aggregate household expenditure in the economy. This method of dealing with the fact that aggregate exposure does not equal zero in the estimation sample is different than the approach taken by Auclert. He assumes the residual exposure is distributed equally across households in the sample. By making use of the national accounts, we believe we are able to get a better handle on the likely MPXs to attach to this residual exposure.

The assumptions we make about the MPX of the young and the old, as well as out of indirectly held URE and NNP exposures, are shown in table G.2. In each case we believe we have made conservative choices that will underestimate the size of the interest rate exposure channel of monetary policy. For the young we choose an MPX of 0.5, in line with the rest of the population. As the young have aggregate negative exposures, choosing an MPX on the low side is conservative. Similarly, for the old we choose an MPX of 0.5, which is on the high side for this age group. The assumption that there is no heterogeneity in MPX within these groups is also a conservative assumption.

Much of the URE and NNP exposure is not held directly on the balance sheet of households but instead indirectly through pension funds, corporates, and the government. There is significant evidence that the MPX out of shocks to the value of pension wealth, stocks, or the government balance sheet is substantially lower than the MPX.
Table G.2  Aggregating Redistribution Elasticities

<table>
<thead>
<tr>
<th>MPX</th>
<th>NNP</th>
<th>URE</th>
<th>$\varepsilon_D$ component</th>
<th>$\varepsilon_R$ component</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sample</td>
<td>See Distribution</td>
<td>-204</td>
<td>-61</td>
<td>-0.82</td>
</tr>
<tr>
<td>Young</td>
<td>0.5</td>
<td>-32</td>
<td>-15</td>
<td>-0.12</td>
</tr>
<tr>
<td>Old</td>
<td>0.5</td>
<td>-23</td>
<td>6</td>
<td>-0.09</td>
</tr>
<tr>
<td>Pension Funds</td>
<td>0.1</td>
<td>137</td>
<td>37</td>
<td>0.10</td>
</tr>
<tr>
<td>Government</td>
<td>0.0</td>
<td>-85</td>
<td>-23</td>
<td>0.00</td>
</tr>
<tr>
<td>Non-financial Corp.</td>
<td>0.1</td>
<td>-49</td>
<td>-13</td>
<td>-0.04</td>
</tr>
<tr>
<td>Financial Sector</td>
<td>0.1</td>
<td>223</td>
<td>61</td>
<td>0.17</td>
</tr>
<tr>
<td>Rest of World</td>
<td>0.0</td>
<td>33</td>
<td>9</td>
<td>0.00</td>
</tr>
<tr>
<td>Total</td>
<td>0</td>
<td>0</td>
<td>-0.80</td>
<td>-0.29</td>
</tr>
</tbody>
</table>

Notes: NNP and URE numbers are in billions of 2015 USD. Pension Funds includes special saving such as children’s savings accounts. See appendix H for detail.

from income. We choose to use the estimate from Maggio, Kermani, and Majlesi (2019) that households’ MPX from changes in stock market wealth is about 10%. This choice is the most quantitatively important for the sufficient statistics, as the bin containing the most exposure is the financial sector, which is positively exposed to interest rate increases. This positive interest rate exposure may seem surprising because banks are typically thought to have long-term assets and short-term debt that would result in negative URE exposure. However, our findings are in line with those of Landier, Sraer, and Thesmar (2013), who find that the financial sector benefits from interest rate hikes overall. An important caveat is due here: we focus on the MPX out of changes in the assets indirectly held by households through the financial sector and do not assume any spending or lending response at the bank level. This assumption may be reasonable in good times when banks are not credit constrained, but may not hold during a banking crisis. Financial frictions could possibly result in monetary policy being much less effective during a banking crisis as the interest rate exposure channel to household spending is counterbalanced by a channel from bank balance sheet interest rate exposure to lending.\(^{14}\)

We choose an MPX of zero for government and the rest of the world. There is no evidence that households respond in any significant way to changes in the government’s balance sheet, and furthermore a low MPX is a conservative assumption for the size of the heterogeneous agent channels. As Denmark is a small part of the world economy, we assume that foreigners spend a negligible proportion of their wealth there.

\(^{14}\)It should be noted that our analysis is all on the household side. Evidence suggests that firms are also sensitive to changes in cash flow; for example, see Blanchard, Lopez-de Silanes, and Shleifer (1994).
G.1 United States

A similar procedure is followed for the United States, except that the MPX for all age groups is estimated using liquid wealth, and URE and NNP exposure not directly held by households is associated with an MPX of 0.1.

H Details on the Calculation of NNP and URE

The Net Nominal Position (NNP) and Unhedged Interest Rate Exposure (URE) for the various sectors in the Danish economy are calculated from our household-level dataset as well as the financial accounts from the national accounts statistics. All calculations are based on average values over the years 2009 to 2015, deflated by the consumer price index.

H.1 NNP and URE for Households

The NNP for households is calculated as financial assets minus liabilities. As financial assets, we include bank deposits as well as the market value of securities (excluding shares). Liabilities include all debt to financial institutions (including credit card debt) as well as publicly administered student debt, tax debt and other debt to government bodies. These data are reported to the tax authorities by financial institutions on behalf of the households.

URE is calculated as annual savings (i.e. after-tax income minus expenditure) plus maturing assets minus maturing liabilities. As maturing assets, we include all bank deposits, thereby assuming that they are floating rate. We assume a maturity of five years for securities held by households and therefore include 20% of the value of securities. Regarding liabilities, we assume that all bank debt is floating rate. According to the interest rate statistics collected by Danmarks Nationalbank since 2013, on average 95% of bank debt from households is floating rate, most of which is tied either to a market reference rate or to the Danmarks Nationalbank rate on certificates of deposit, with immediate adjustment. For mortgage debt, we have detailed information allowing us to calculate the stock of debt which is due to have interest rates reset over the coming 12 months, and assume that the new rate will only apply for half of the year. Voluntary refinancing of mortgage loans, with or without extraction of additional equity, takes place to a large extent in Denmark. Our measure of maturing liabilities only includes the loans that are contractually due to have their interest rates reset, and we do not attempt to
estimate the amount of additional refinancing. For remaining liabilities, which constitute small amounts, we have no information regarding maturity, so we assume five years.

H.2 Other Sectors

NNP for the other sectors in the economy is obtained from the financial accounts statistics compiled by Danmarks Nationalbank. To most closely resemble the definition used in the household-level data, we define NNP as net assets (i.e., assets minus liabilities) in the following categories: "Currency and deposits", "Securities other than shares", "Loans", and "Trade credits and other accounts receivable/payable".

NNP for the whole economy should, in principle, sum to 0. However, the household-level microdata on bank deposits that we have access to is exclusive of certain types of savings (specialized children’s savings accounts as well as some forms of pension savings accounts administered by banks), which are included in the financial accounts statistics. For the age group included in our sample, these types of accounts can be assumed to be largely illiquid. We therefore group those deposits (33 billion USD) together with the assets of pension funds (see table G.2).15

URE for non-households is also based on the financial accounts. In the national accounts, we do not observe the maturity of different asset and liability classes. We hold household URE fixed at the values from the micro-level data and take advantage of the identity that total URE in the economy must be 0 to calibrate the maturity for the remaining sectors of the economy. This results in a maturity of assets and liabilities for non-households of 3.65 years.

I Persistent Consumption Response

Our estimation procedure makes the assumption that the consumption response to a transitory income shock decays to zero in a period of two years or less. A slower decay will lead to a downward bias in our estimates of the transitory MPX. Figure I.1 shows the results of our estimation procedure on simulated data under two different assumptions about the transitory consumption response.

15In practice, this amount is calculated as a residual, which may also reflect other minor differences between the household-level data and the national accounts statistics. For example, holdings of banknotes and coins are not observed in the microdata but are allocated based on certain assumptions in the financial accounts. For our exercise, the impact of such other differences is likely to be small.
The exponential decay line assumes that the consumption flow following a transitory shock decays exponentially.\textsuperscript{16} We vary the decay rate to match a range of year 1 MPCs and assume that the entire transitory income is eventually consumed. For high MPCs, and especially those over 0.5, there is little bias. However, for MPCs significantly below 0.5 our method results in downward-biased estimates. This bias arises because low MPCs, combined with exponential consumption decay, result in a relatively stable consumption flow over the first few years that has not declined close to zero after two years.

Empirical evidence suggests that in fact the consumption response to a transitory shock decays quickly in the first few months and then more slowly after that.\textsuperscript{17} The “Fagereng et al.” line in figure I.1 shows the MPC estimate in simulated data in which the consumption response decays according to the estimates made in Fagereng, Holm, and Natvik (2021). In this case, the fast decay in the first few months results in a smaller bias than the exponential case for low MPCs, while the fact that the decay is slower following these first months results in a larger bias for high MPCs. Overall it seems likely that our assumption about the persistence of the consumption response leads to a slight downward bias across the range of MPCs.

We also show that our MPX estimates are not very sensitive to the choice of $N$ (years of growth in our identification equations) between 3 and 6, which lends further support to the fact that assuming a two-year limit does not bias our results too much.\textsuperscript{18}

\subsection*{I.1 Details on the simulations}

For the simulations we divided each year into 20 sub-intervals. Both permanent and transitory shocks occur each period, and the transitory shocks have no persistence. At an annual frequency the variance of permanent and transitory shocks are equal. Households spend their permanent income each period, along with their consumption response to the history of transitory shocks. For the exponential decay model, this is

$$c_t = p_t + (1 - \rho) \sum_{n=0}^{\infty} \rho^n \epsilon_{t-n}$$

In Fagereng, Holm, and Natvik (2021) the T year MPC is estimated as a function:

$$\text{MPC}_T = \theta_1 T^{\theta_2}$$

\textsuperscript{16}Standard buffer-stock models give rise to a consumption response that decays close to exponentially.
\textsuperscript{17}Both Fagereng, Holm, and Natvik (2021) and Gelman (2021) provide evidence for this.
\textsuperscript{18}Using $N$ equal to 4 and 5 instead of 3, 4, and 5 allows us to extend the consumption response out to three years, at the expense of losing data and becoming more sensitive to misspecification of the income process.
where $\theta_1$ controls the size of the response and $\theta_2$ the speed of decay. We vary $\theta_1$ and choose $\theta_2 = 0.2142$ according to their estimate. In this model consumption in period $t$ (measured in sub-intervals) is:

$$c_t = p_t + \theta_1 \sum_{n=0}^{\infty} \left( \left( \frac{n+1}{20} \right)^{\theta_2} - \left( \frac{n}{20} \right)^{\theta_2} \right) \varepsilon_{t-n}$$

We then time aggregate both income and consumption over each 20-sub-interval period, choose a sample of 13 years, and run our estimation procedure with $N = 3, 4, 5$. The transitory MPC estimates are shown in figure I.1, and the permanent estimates are shown in figure I.2. The bias in permanent estimates is small across the range of transitory MPCs.

### I.2 Estimates Using Different Values of $N$

#### Table I.1 $\psi$ Estimates Using Different $N$
Figure I.2 Bias from Persistent Consumption

<table>
<thead>
<tr>
<th></th>
<th>$n_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.55</td>
</tr>
<tr>
<td>2</td>
<td>0.62</td>
</tr>
<tr>
<td>3</td>
<td>0.63</td>
</tr>
<tr>
<td>4</td>
<td>0.67</td>
</tr>
<tr>
<td>5</td>
<td>0.65</td>
</tr>
<tr>
<td>6</td>
<td></td>
</tr>
</tbody>
</table>

Table I.1 shows the estimates of the transitory MPX that we recover from our estimation sample when we just use $N = n_1, n_2$ in our identification equations 3 and 5. Remember in our main results we used GMM with $N = 3, 4, 5$ and we have circled $N = 3, 5$ to highlight where we get identification from in the paper. The purpose of this exercise is to show that the estimation results are not very sensitive to the values of $N$ chosen, providing more evidence that the assumption we made that the transitory consumption response lasts less than two years is not biasing our results significantly. In fact, the results are not changed dramatically even when $N = 1, 2$, which suggests the majority of the transitory consumption response is short-lived.
J RIP or HIP Income Process?

J.1 RIP or HIP Income Process?

Our method makes strong assumptions on the income process—namely, that there is no persistent idiosyncratic component to income growth and that the process contains a random walk. Guvenen (2009) shows that it is empirically difficult to distinguish between a ‘Restricted Income Profile’ (RIP) like this and a ‘Heterogenous Income Profile’ (HIP) income process, in which (i) shocks to income are much less persistent (e.g., AR(1) with \( \rho \approx 0.8 \)), and (ii) households have a persistent idiosyncratic growth component. The reason the RIP and HIP processes are difficult to tell apart is that the two features (i) and (ii) act in opposite directions on the cross-section variance of income growth. The less persistent income shocks lead the cross-sectional income growth variance to not grow as fast as the HIP model, while the persistent idiosyncratic growth component leads the same variance to grow at a faster rate. The result is that the increase in variance of income growth over three to four years is approximately the same as the increase from four to five years. To the extent that the consumption response to these semi-permanent shocks is similar to the response to the idiosyncratic persistent growth component, our methodology will continue to provide reasonable estimates of the “permanent” MPX and the more familiar transitory MPX.

Both the Restricted Income Profile (RIP) and Heterogeneous Income Profile (HIP) processes can be described by the equations:

\[
y^i_h = \beta^i + z^i_h + \varepsilon^i_h \\
z^i_h = \rho z^i_{h-1} + \eta^i_h
\]

where \( i \) indexes the worker and \( h \) the years of experience. \( \varepsilon^i_h \) represents a transitory shock to income, while \( \eta^i_h \) is persistent. \( \beta^i \) represents an idiosyncratic persistent growth factor.

In the RIP model, \( \beta^i = 0 \) and \( \rho \) is usually estimated to be close to 1 (in this paper we assumed \( \rho = 1 \)). In the HIP model, \( \beta^i \) has a cross-sectional variance \( \sigma^2_\beta > 0 \), and \( \rho \) is normally estimated to be significantly lower than 1, around 0.8. The reason these are difficult to tell apart is because the theory does not give a strong indication in which model the cross-sectional variance of income growth over \( N \) years should grow.

---

\(^{19}\)See Guvenen (2007) for an example of why this might be the case: if households do not know their own idiosyncratic growth ex-ante, a Bayesian learning process will be slow, so households (at least initially) will react in similar ways to changes in income due to this persistent growth component as a true income shock.
faster. In the RIP model with $\rho = 1$, the cross-sectional variance of income growth increases linearly with $N$. In the RIP model with $\rho \approx 0.8$, the growth in the cross-sectional variance of income growth will decrease due to the low $\rho$ but increase due to the idiosyncratic $\beta^i$.

Figure J.1 shows the empirical values for income growth variance and the covariance of income and expenditure growth over $N$ years. We have also plotted the fitted values for these statistics that are implied by our model when fitted to $N = 3, 4, 5$ as we do in the paper. We see the empirical variance and covariance decline slightly below the model fitted line as $N$ becomes large, which fits with the finding that $\rho$ in the RIP model is usually slightly below 1.0, around 0.98 or 0.99. We also note that around the region where we achieve our identification ($N = 3, 4, 5$), there is little curvature in the empirical statistics, and the increase in both variance and covariance is close to linear.

While this linearity around $N = 3, 4, 5$ cannot help us distinguish between the RIP and HIP process, it does imply that our empirical methodology may be somewhat robust to misspecification along this dimension. If we assume that the expenditure response to a change in $z^i_h$ and to the increase from the persistent idiosyncratic growth are equal to $\phi$, and the response to a transitory shock is $\psi$, that is:

$$\Delta^N c_h^i \approx \phi \Delta^N (\beta^i h + z^i_h) + \psi \Delta^N \varepsilon^i_h$$

Then, the fact that $\text{Var}(\Delta^N (\beta^i h + z^i_h))$ grows approximately linearly with $N$ means that our empirical method will correctly identify $\phi$ and $\psi$.

A full investigation of the implications of different income processes is beyond the scope of this paper but would be a useful exercise for future research.

K Time-Varying Risk

We have assumed that idiosyncratic risk remains constant over time. Given that our sample period covers the great recession, this may not be appropriate. Here we show how the variance of income growth has varied over time, peaking just after the crisis in 2010. In order to test how much this time-varying risk might bias our results, we simulated data with $\phi = 1$ and $\psi = 0.5$, with permanent variance equal to estimates from the data and transitory variance varying in order to match the time-varying income risk pattern observed in the data. When we run this simulation we find estimates of $\phi$ and $\psi$ within 1% of their true values.

Figure K.1 shows how the standard deviation of income growth has changed over the
sample period. From trough to peak, the standard deviation increases approximately 10%. In the simulation referred to in section K, we assume that both transitory income and transitory consumption response have no persistence. We divide each year into 20 sub-periods, choose the variance of permanent shocks to be 0.003, and allow time-varying transitory shocks to match the pattern in figure K.1. We choose values of $\phi = 1$ and $\psi = 0.5$ and apply our estimation procedure (that assumes constant variance) to the simulated data. We recover estimated values of $\phi$ and $\psi$ to be 1.006 and 0.499, respectively.

L Robustness

L.1 Measurement Error

Our identification comes from estimating $\text{Var}(\Delta^N \bar{y})$ and $\text{Cov}(\Delta^N \bar{c}, \Delta^N \bar{y})$ using our observed data. For unbiased estimates of $\text{Var}(\Delta^N \bar{y})$ we require no measurement error in our observed changes in labor income. For unbiased estimation of $\text{Cov}(\Delta^N \bar{c}, \Delta^N \bar{y})$ we only require (further to no measurement error in income growth) that the measurement error in expenditure growth is uncorrelated with labor income growth. As our expenditure is imputed from income and changes in assets, this is potentially more of a concern than would be the case in survey data in which questions about consumption are not directly linked to those on income. We will examine potential sources of error in labor income and imputed consumption.
L.1.1 Labor Income

For most workers, labor income is well measured. Third party reporting, along with a high level of trust in government institutions, means that underreporting is likely low. The black economy in Denmark is small, and to the extent that any growth in unreported income is uncorrelated with growth in reported income this will not bias our estimates. In contrast to survey data, in which measurement error in income is likely to downwardly bias transitory MPX estimates, this is of little concern in our data.

L.1.2 Imputed Expenditure

Expenditure is calculated as the residual of total household income (including interest and dividends) after pension contributions and the change in net wealth have been deducted. For households with simple financial lives (which we believe fits most of the Danish population), this should work well. There are a few scenarios that merit further investigation.

- Stock Market Capital Gains: Only 10% of Danish households directly own stocks or mutual funds. In online appendix L we show that the qualitative patterns we observe are unchanged even when we completely remove these households from the sample. For households that do own stocks, we assume the return they

\[\text{\footnotesize 20Such income may show up as a change in net wealth and hence expenditure, but measurement error in the change in expenditure uncorrelated with the change in labor income will not bias our MPX estimates.}\]

\[\text{\footnotesize 21In our calculation we directly observe flows in and out of pension accounts, so these can be treated as off balance sheet in which capital gains do not affect our expenditure calculation.}\]
receive is equal to a diversified portfolio of Danish stocks. Given that different households will have their own idiosyncratic portfolios, this methodology will result in significant measurement error. Baker, Kueng, Pagel, and Meyer (2018) show that the size of this measurement error is not only correlated with income and wealth, but also with the business cycle. Furthermore, Fagereng, Guiso, Malacrino, and Pistaferri (2019) show that some groups of investors consistently outperform the market, which would lead us to consistently underestimate their expenditure. Our concern, however, is that the change in measurement error of expenditure be correlated with the change in labor income. Consistently underestimating expenditure by the same amount is therefore not a problem for us. Furthermore, as we have removed all aggregate effects from the labor income residuals that we use in estimation, any measurement error correlated with the business cycle will be uncorrelated with our measure of changes in labor income. We see two potential ways in which mis-measuring stock returns may bias our results. First, if households have significantly invested in the stock of the firm they work for, which is likely only to be the case for high-level management. Second, to the extent that households invest their labor income gains halfway through the year, we will underestimate expenditure for those whose income increases, and overestimate it for those whose income decreases, leading us to underestimate the MPX. The size of this bias is limited by the size of excess expected returns, so our MPX estimate will be biased by no more than a few percentage points.

**Family and Friends Transfers:** If a household receives a transfer of money from their parents, for example, imputed expenditure will be lower than true expenditure by this amount. Large transfers typically occur upon death of a parent, which is likely to be uncorrelated with the household head’s labor income, or when purchasing a house—years that we have already excluded from our sample. However, to the extent that friends and family actively insure each other’s labor income, our MPX estimates will be upward biased.

**Off-Balance-Sheet Assets:** A larger concern is that some forms of saving may be hidden off balance sheet. Our imputation method would interpret off-balance-sheet saving as expenditure, so our estimate of the MPX would increase one-to-one for each percentage point of saving out of income shocks performed off balance sheet. All Danish banks and brokers are required to report their clients’ holdings, so off-balance-sheet assets are likely to be either offshore or nonfinancial assets.
Such off-balance-sheet saving would be a large concern if we were focused on the expenditure of the super wealthy 0.1%, but is less so when dividing the population into quintiles or deciles as we have done.

As would be clear from the main text, we have made a number of choices regarding both data and variable definitions as well as more methodological issues. In a series of graphs, this appendix presents a number of robustness checks aimed at assessing the extent to which our results are sensitive to the specific choices.

We begin with a number of robustness checks regarding our imputed expenditure measure, which may suffer from measurement error. In figure L.2, we compare our baseline estimates of the MPX to estimates based on different sample selection procedures. First, we exclude all households that own stocks corresponding to more than 10,000 USD (10% of households in our sample). Second, we do not remove households that have negative imputed expenditure. We remove those households in our baseline sample because negative expenditure is clearly not a good estimate of actual expenditure. However, for example, in the event that negative expenditure arises because of classical measurement error, removal of negative estimates may be asymmetric and introduce an upward bias in average imputed expenditure. Third, to check that large outliers do not drive our results, we remove observations in the top and bottom 2.5% in terms of level and change of income and expenditure. In the baseline calculations, we use only a 1% cutoff. Our results are qualitatively unchanged when using these alternative approaches to take account of measurement error. In terms of magnitudes of the estimated MPXs, the largest difference to the baseline results seems to be found when we include negative expenditure estimates. As expected, this makes the largest difference
Figure L.2 Robustness of Liquid Wealth and URE Distributions

among the wealthier households. The specification of outliers also matters somewhat for the point estimates of MPX in certain groups of households, but differences are not large.

Another robustness check consists of specifying consumption and income in logs rather than in levels. The fundamental difference is that the log specification yields an elasticity rather than an MPX. Hence, some difference between level and log results must be expected for households that only spend a fraction of their annual income (typically wealthier households). Indeed, as expected, figure L.3 demonstrates that results hold qualitatively when specifying income and expenditure in logs rather than in levels, whereas estimated elasticities are higher than the MPXs for the wealthier households and those with high URE. Time-varying income risk may also potentially contribute to differences between results based on levels and logs. However, as shown in section K, this is not likely to be important in our setting.

As discussed in section 3.1, we use total household income as our prime measure of income in line with previous consumption literature. The literature on idiosyncratic income processes tends to use income of the head of the household. Various mechanisms—e.g., intra-household income insurance—may give rise to differences between results based on income of the head of household and total household income. However, figure L.4 demonstrates that there is virtually no difference in our results between using total household income and only the household head’s income. Online appendix Q briefly
discusses the potential role that intra-household insurance may play, which we leave as an area for future research.

Finally, figure II shows the distribution of MPX by quintile of liquid wealth. It might be argued that the relevant level of liquid wealth is relative to income rather than in absolute terms. Figure L.5 demonstrates that results based on quintiles of liquid wealth divided by permanent income are similar. Also, results (not shown here) where deciles are based on a broader definition of liquid wealth—i.e., including stock and bond holdings—are similar to our baseline results.

M Durables

A critique of our empirical methodology is that it does not take account of durable goods, while our data include all spending (except on real estate) and therefore include large and durable goods such as cars and home improvements. The empirical model assumes that in response to a transitive income shock, expenditure increases temporarily for up to two years, which is entirely consistent with a model that includes durable goods. However, the model assumes that in response to a permanent shock to income, expenditure increases once to a new permanent level. A model that included durable goods would instead imply a large one-off expenditure on durable goods to get the households up to their desired flow of durable good services, followed by a decrease
Figure L.4  Results Using Total Labor Income and Head Labor Income

Figure L.5  Results Using Quintiles of Liquid Wealth over Permanent Income vs Liquid Wealth
back to a permanent level of spending that accounts for replenishing the higher level of depreciating durable goods.

We address this problem in two ways. First we show that our MPX estimates are unbiased in a simple model that includes durables, as long as we interpret the MPX out of transitory shocks to include durable expenditure (the correct definition for understanding aggregate demand) and the MPX out of permanent shocks to include only the consumption flow from durables. Second, we are able to construct a nondurable consumption proxy for each household using registry data on car purchases. This proxy has large measurement error, but will result in unbiased estimates of the MPC (excluding durables) to both permanent and transitory shocks. The estimated MPCs by liquid wealth quintile are shown in figure M.1. The figure shows the estimates using the nondurable proxy are, as expected, lower than those including all expenditures, although the change in magnitude is similar in size to the overall fraction of durable expenditure, suggesting durables do not play a special role in expenditures following transitory shocks. For the top quintile, durables do appear to play an outsized role, accounting for about a third of the expenditure response to transitory shocks.
M.1 Modeling Durables

It will help to write down a simple model. The model will show that our empirical methodology continues to estimate the consumption response to permanent and transitory shocks, but that these need to be interpreted carefully. The model uses the same income process as section 2.1. Remembering the income process is made up of two martingale processes, \( P_t \) and \( Q_t \), which may have jumps, instantaneous income is given by

\[
dy_t = \left( \int_0^t dP_s \right) dt + dQ_t
\]

while instantaneous expenditure now has both a durable and a nondurable component:

\[
dc_t = \phi_{nd} \left( \int_0^t dP_s \right) dt + \phi_d dP_t + \psi dQ_s
\]

Here we have assumed that the expenditure response to transitory shocks is instantaneous, but it would not change things to assume as before that the response decays to zero after two years. However, it is important that the durable component of the expenditure response to permanent shocks occurs instantaneously with the shock (or very soon after). Aggregating income and consumption annually gives

\[
\Delta^N \bar{y}_T = \left( \int_{T-N-1}^{T-N} (s - (T - N - 1)) dP_s + \int_{T-N}^{T-N-1} dP_s + \int_{T-N}^{T} (T - s) dP_s \right) \\
+ \left( \int_{T-1}^{T} dQ_t - \int_{T-N-1}^{T-N} dQ_t \right)
\]

\[
\Delta^N \bar{c}_T = \phi_{nd} \left( \int_{T-N-1}^{T-N} (s - (T - N - 1)) dP_s + \int_{T-N}^{T-N-1} dP_s + \int_{T-N}^{T} (T - s) dP_s \right) \\
+ \phi_d \left( \int_{T-1}^{T} dP_t - \int_{T-N-1}^{T-N} dP_t \right) \\
+ \psi \left( \int_{T-1}^{T} dQ_t - \int_{T-N-1}^{T-N} dQ_t \right)
\]

From this we can calculate the covariance:

\[
\text{Cov}(\Delta^N \bar{c}_T, \Delta^N \bar{y}_T) = \phi_{nd} \text{Var}(\Delta^N \bar{y}_T) \\
+ \phi_d \left( \int_{T-1}^{T} (T - s) \sigma^2_{P_t} dt - \int_{T-N-1}^{T-N} (s - (T - N - 1)) \sigma^2_{P_t} dt \right)
\]

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So the durable component of the covariance cancels out, and our identification method correctly identifies $\phi_{nd}$ and $\psi$ but is unable to identify $\phi_d$.

However, if there is some delay between the household receiving the permanent income shock and purchasing the durable goods, then this introduces an upward bias into the estimate of transitory MPX. The size of the bias grows with the number of months delay between the permanent income shock and the durable goods purchase, plateauing after 12 months at a level of $\frac{\sigma_p^2}{2\sigma_q^2}$ $\phi_d$. Figure M.1 shows how this bias increases with the delay.

In order to quantify how large this bias may be in practice, we make use of the car registry data available in Denmark. Using data on the current value of cars owned by a household, we perform the same residual calculation to find the change in car value that is unpredictable with the household characteristics we are able to observe. We then construct two new expenditure panels: one in which we remove expenditures on cars, and one in which we make a proxy for non-durable consumption by removing expenditures on cars multiplied by $\frac{1}{0.421}$ (car purchases make up 42.1% of durable expenditure in Denmark):

$$C_{T\text{nocar}} = C_T - \Delta CarValue$$

$$C_{T\text{nondurable}} = C_T - \frac{1}{0.421} \Delta CarValue$$

The second, nondurable proxy consumption panel, can be modeled as the true non-durable consumption panel with classical measurement error added. This classical
measurement error does not bias our estimates, so we can use this nondurable proxy panel to estimate an unbiased MPC out of transitory shocks, where the MPC does not include durable expenditures.

The results of this exercise can be seen in figure M.1. Even without bias, we would expect the nondurable proxy estimates to be lower than those including all expenditures, as the definition of transitory MPX changes over the three panels to exclude cars and then all durable goods. For the lower quintiles of liquid wealth it therefore looks as though the bias is likely small, as nondurable goods make up 10% of spending and the MPX estimates are smaller by approximately 10% in this region. For the top quintile of liquid wealth there seems to be some bias, with the estimate of MPX for all expenditures decreasing from 25% to an MPC for nondurable goods of 17%.

While there is some evidence that our results may be biased upward for those in the top quintiles of liquid assets, this bias will only have a small effect on our overall conclusions. As the relevant number for the monetary policy exercise is the MPX rather than the MPC, we have chosen not to adjust our baseline results using this method and accept that a small bias may exist in our data. It should be noted that such a bias will cause the heterogeneous channels of monetary policy to appear smaller than they actually are.

N Interpolating U.S. MPX

As we discussed in section 5.2, liquid wealth holdings among U.S. households is lower than for Danish households, in absolute value and especially as a ratio of income. In our baseline results, we interpolate the MPX for U.S. households from Danish households according to the percentile of liquid wealth holding they are in. For example, a household in the 20th percentile of liquid wealth holdings in the United States is allocated the same MPX as a household in the 20th percentile of liquid wealth holdings in Denmark.

In this section we take two alternative approaches. The first is to interpolate based on the absolute level of liquid wealth. That is, a US. household with $2,000 of liquid wealth is allocated the same MPX as a Danish household with $2,000 of liquid wealth (equivalent in DKK). The second is to interpolate based on the ratio of liquid wealth to income.

The sufficient statistics under the three interpolation methods are shown in table N.1. This lower levels of liquid wealth and liquid wealth to income in the United States result in somewhat higher estimates of $M$, the income-weighted MPX, for the two alternative
interpolation methods. Qualitatively, the results for the redistribution elasticities are
the same as in the baseline, and the estimate for the interest rate exposure channel, $\varepsilon_R$
is little changed.

### O MPX Heterogeneity over Liquid Wealth and Income

In this appendix we divide the population up into quintiles along two dimensions: liquid
wealth and income. The step pattern of figure O.1, in which MPX steps down along the
liquid wealth quintile, but is flat along the income quintile, demonstrates our finding
that conditional on liquid wealth, the other dimensions we explore, such as income, have
no further predictive power on MPX levels.

### P Distribution of Permanent MPX by NNP, URE, and Income

Figure P.1 shows the distribution of both transitory and permanent MPX by NNP, URE
and income decile. The transitory numbers are a repeat of figure V.

### Q Intra-household Income Insurance

As discussed in section 3.1, we use labor income of the head of the household as our
prime measure of income, in line with previous literature. Figure L.4 demonstrates
that results based on total household income and income of the head of household are
similar. However, MPXs from transitory shocks to the income of the spouse are lower
than MPXs from shocks to total income, in particular for the less wealthy households, as demonstrated in figure Q.1. This indicates heterogeneity in the role that intra-household income insurance plays across different groups of households. We leave this interesting topic for future research.

**Online Appendix References**


Figure P.1  MPX Distribution by URE, NNP, and Income
**Figure Q.1** Results Using Total, Head, and Spouse Labor Income


