This appendix computes dynamics and steady state of the square of the idiosyncratic component of permanent income (from which the variance can be derived).

$$
\begin{aligned}
p_{t+1,i} &= p_{t,i}\psi_{t+1,i}\omega_{t+1,i} + (1-\omega_{t+1,i}) && (1)\\
p_{t+1,i}^2 &= (p_{t,i}\psi_{t+1,i}\omega_{t+1,i})^2 + 2p_{t,i}\psi_{t+1,i}\underbrace{(1-\omega_{t+1,i})\omega_{t+1,i}}_{\equiv 0} + (1-\omega_{t+1,i})^2 && (2)
\end{aligned}
$$

and since $\mathbb{E}_t[(1-\omega_{t+1,i})^2] = \mathbb{E}_t[(1-2\omega_{t+1,i}+\omega_{t+1,i}^2)] = (1-\Omega)$ we have

$$
\begin{aligned}
\mathbb{E}_t[p_{t+1,i}^2] &= \mathbb{E}_t[(p_{t,i}\psi_{t+1,i}\omega_{t+1,i})^2] + (1-\Omega) && (3)\\
&= p_{t,i}^2\Omega\mathbb{E}[\psi^2] + (1-\Omega) && (4)
\end{aligned}
$$

so defining the mean operator $\mathbb{M}[\bullet_t] = \int_0^1 \bullet_{t,\iota} d\iota$, we have

$$
\mathbb{M}\left[p_{t+1}^2\right] = \mathbb{M}[p_t^2]\Omega\mathbb{E}[\psi^2] + (1-\Omega) \qquad (5)
$$

so that the steady state level of $\mathbb{M}[p^2] \equiv \lim_{t\to\infty}\mathbb{M}[p_t^2]$ can be found from

$$
\begin{aligned}
\mathbb{M}[p^2] &= (1-\Omega) + \Omega\mathbb{E}[\psi^2]\mathbb{M}[p^2] && (6)\\
\mathbb{M}[p^2] &= \left(\frac{1-\Omega}{1-\Omega\mathbb{E}[\psi^2]}\right) && (7)
\end{aligned}
$$

Finally, note the relation between p^2 and the variance of p:

$$
\begin{aligned}
\sigma_p^2 &= \mathbb{M}[(p-\mathbb{M}[p])^2] && (8)\\
&= \mathbb{M}[(p^2 - 2p\mathbb{M}[p] + (\mathbb{M}[p])^2)] && (9)\\
&= \mathbb{M}[p^2] - 1 && (10)
\end{aligned}
$$

where the last line follows because under the other assumptions we have made, $\mathbb{M}[p] = 1$.

Of course for the preceding derivations to be valid, it is necessary to impose the parameter restriction $\Omega\mathbb{E}[\psi^2] < 1$. This requires that income does not spread out so quickly among consumers who survive as to overcome the compression of the distribution that arises because of death.